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Improving the Delaunay tessellation particle tracking algorithm in the three-dimensional field

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ABSTRACT

Particle tracking velocimetry (PTV) algorithms match particles across two consecutive frames corresponding to a certain flow pattern. PTV based on Delaunay tessellation (DT-PTV) has a succinct structure and depends minimally on algorithmic assumptions. This study proposes several methods for improving DT-PTV performance in the three-dimensional field. The improved version, called 3DT-PTV, is tested using synthetic flows with various parameters and under difficult circumstances. Results show that 3DT-PTV performs better than the classical version in addressing flows with a noticeable ratio of particles without match and those with an excessive ratio of the inter-frame particle displacement to the inter-particle distance.

1. Introduction

Particle imaging techniques have developed rapidly over the past 20 years because of improvements in experimental capabilities. One of the most important contemporary applications of imaging techniques is tracking individual particles in three-dimensional (3-D) flows to allow statistical analysis of particle motion. This task is impossible in classical particle image velocimetry (PIV) correlation algorithms which are based on the Eulerian concept [1], whereas it is solvable with the particle tracking velocimetry (PTV) algorithms which are based on the Lagrangian concept. Hardware experimentation is developed from the evaluation of the velocity component in the depth direction [2,3] to the identification of instantaneous 3-D particle locations [4–7], which serves as the input [7] for the particle tracking PTV algorithm.

A PTV algorithm is developed to match the 3-D particle locations from two consecutive frames corresponding to the continuous motion of particle groups. The algorithm is derived from the PTV based on Delaunay tessellation (DT-PTV) proposed by Song et al. [8], which is characterized by structural simplicity and the capability to handle flows with strong rotation. Firstly, the pattern and algorithmic assumptions, or heuristics, for PTV are introduced to better interpret DT-PTV.

The PTV pattern is either a geometric structure comprising particles in the same frame [9,10], a self-defined index representing such structure [11,12] or a mathematical combination of all possible particle matches [13,14]. PTV derives the final particle matches from the inter-frame pattern matches which are fulfilled based on the geometric or logical pattern similarity criteria. Therefore, pattern matching is called first-level matching. Baek and Lee [15] defined the heuristics for PTV. When considering the two-frame PTV involving only one kind of 'velocity' (the distance between two matched particles over the time interval between two frames), then the heuristics can be simplified as shown in Fig. 1. Heuristic 1: the inter-frame displacement of the individual particle is limited, providing the basis for candidate selection in first-level matching. Heuristic 2: the displacements of particles within the local range are quasi-parallel, describing the local coherent structure. Heuristic 3: the match should be logical, actually functioning as a request for removing spurious matches.

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Nomenclature

Symbols

A, B, C, D, E, F	constant parameters in PTV with the relaxation method (NRX)
C_f	tuning parameter in homemade flow
C_{ij}	similarity between two patterns
D_i	targeted triangle pattern in the first frame
d_m	mean inter-particle distance
d_{max}	maximum inter-particle distance
f_m	mean inter-frame particle displacement
f_{max}	presumed maximum inter-frame particle displacement
f'_{max}	actual maximum inter-frame particle displacement
M_c	number of elements in Θ_c
M_d	particle number in homemade flow
M_μ	number of different values for the noise ratio μ
P_i	probability of x_i without match
P_{ij}	probability of x_i matching y_j
Q_k	number of reference particles for x_k
q_{Xi}	net flux through pattern X_i
R_c	radius of the search area for candidate particles
R_r	radius of the search area for reference particles
S	computation of the area
$\boldsymbol{v}$	velocity vector representing the matching relationship
X_i	pattern in the first frame
X'_i	identifying (ID) vector of X_i
x_i	particle in the first frame
x_k	reference particle for x_i
Y_j	pattern in the second frame
Y'_j	identifying (ID) vector of Y_j
y_j	particle in the second frame
y_l	reference particle for y_j
Z_k	number of all candidate particles for all reference particles in Θ_r

Greek characters

α	candidate marking number
η	accuracy of PTV
Θ_c	set of candidate particles for x_i
Θ_r	set of reference particles for x_i
θ	non-uniformity of particle distribution
μ	noise ratio
ω	noise-weighted average failure ratio of PTV

Superscripts

(n)	index of iteration step
$(*)$	non-normalized probability value

DT-PTV is a classical cluster-based matching algorithm, and its pattern is the Delaunay triangle in the two-dimensional (2-D) field (or tetrahedron in the 3-D field), which comprises neighboring particles from the same frame. The DT-PTV algorithm is simple because it only follows Heuristic 1 and disregards Heuristic 2.

2. DT-PTV and its improvements

2.1. Selection of matching candidates

We will explain the classical DT-PTV in the 2-D field. As shown in Fig. 2, a DT grid comprises particles in each of the two consecutive frames. The first step is selecting the matching triangle from the second frame for each triangle in the first frame. Consider that a random triangle D_i is in the first frame (D_i also denotes the center), as shown in Fig. 2a. By adopting Heuristic 1 and the coordinates of D_i, any triangle can be selected as the matching candidate of D_i as long as its center is within the circular search area (Fig. 2b).

2.2. The first improvement

The first improvement is actually a function that allows DT-PTV to abandon the fixed radius generating the search area. Based on Heuristic 1, selecting candidate patterns is no longer determined by the fixed radius but by the DT grid. Firstly, D_i and all pattern centers in the second frame are collected to form a DT grid. In this new DT grid, patterns with centers directly connected to D_i are regarded as candidates in the first layer and marked as '1'. Remain patterns whose centers are directly connected to the centers in the first layer are regarded as in the second layer and marked as '2' (Fig. 3), and so on. Secondly, the final candidates for D_i are determined according to a positive integer α which is named the candidate marking number. Specifically, any candidate marked m or less than m is selected if $\alpha = m$, such that m is a random positive integer.

α replaces the fixed radius in selecting candidates based on Heuristic 1 without the need to presume specific flow parameters because a suitable fixed radius should be no less than the maximum inter-frame particle displacement. Given the mean inter-particle distance d_m and the mean inter-frame particle displacement f_m, then Heuristic 1 can be paraphrased, such that the value of $d_m f_m^{-1}$, which is mainly determined by the frame rate of the camera, must be large enough to allow PTV to work. Under ideal conditions in which all particles are static ($f_m = 0$), the nearest pattern to D_i can simply be found as the matching pattern. Under the worst condition for a fixed volume, $d_m f_m^{-1}$ will become undesirably small because of two possibilities. Firstly, f_m becomes excessively huge, such that the change in particle locations in a frame sequence ceases to inherently describe the continuous change of a certain flow pattern. PTV does not adequately match particles in such flow. That is, PTV is destined to fail. Secondly, d_m becomes extremely small because of the excessive number of particles, such that even at $\alpha = 1$, a small fixed area will include dozens of