Measurement 49 (2014) 148–152

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Special signals in the calibration of systems for measuring dynamic quantities

Krzysztof Tomczyk *

Cracow University of Technology, Faculty of Electrical and Computer Engineering, Poland

A R T I C L E I N F O

Article history:
Received 16 July 2013
Received in revised form 15 October 2013
Accepted 25 November 2013
Available online 4 December 2013

Keywords:
Integral-square error
Signals with one constraint

A B S T R A C T

The determination of errors generated by systems measuring dynamically changeable signals presents a difficult problem due to their unknown shape and the measurement duration. This paper presents a proposed solution to this problem by means of the signals maximizing chosen error criterion. Thus, it refers to the commonly applied methods for determining the accuracy class of systems intended for the measurement of static quantities.

The method of determining a signal with one constraint, maximizing integral-square error, is discussed in this paper in detail. As an example, four acceleration sensors are considered and the maximizing signals from the range of one to twenty-five switchings are determined. It is worth underlining that the paper presents a solution obtained by an analytical method instead of using optimization methods, the application of which is necessary in the case of signals with an increased number of constraints.

1. Introduction

In static measurements, the quality of the obtained results depends on the accuracy class of the measuring instruments [1].

In the case of systems intended for the measurement of dynamic signals, the legal regulations relating to the calibration procedures do not exist. This results from the fact, that errors generated by these systems depend not only on their dynamic properties, but also on the shape of the input signals. Such errors are expressed by the convolution integral of two signals. The first is the input signal, which can achieve quite different shapes. The second signal presents the kernel of the system and this one directly describe the system dynamics [9,10].

Systems measuring the dynamic quantities are usually calibrated in the frequency domain on the basis of their frequency characteristics. Most often the amplitude–frequency characteristic measurement is sufficient in this case [2–4].

One can find the different methods dealing with calibration of the measuring systems, for example, by means of steady state random signals, one harmonic or multi-component signals and so forth [5,6]. However, it just seems that the most important theory of dynamic system calibration is based on the theory of maximum errors [7–12]. According to this theory, a signal of any shape, which could occur on the input of the measuring system will always generate the error of less than or at most equal to the maximum value. This means that the calibration result is completely independent of the shape and dynamics of the input signal [7–9].

The procedures of calibration include:

1. Determination of the mathematical model of the measuring system and its standard.
2. Choice of the error criterion.
3. Analysis of the constraints referring to the input signal.
4. Determination of the maximizing signals.
5. Error calculation.

* Tel.: +48 12 628 25 43
E-mail address: k.tomczyk@cyfronet.pl

0263-2241/$ - see front matter
http://dx.doi.org/10.1016/j.measurement.2013.11.047

The integral-square error, which is very important for many classes of measuring systems, is applied in this paper.

According to the theory of the maximum errors, one constraint, referring to the magnitude a, or simultaneously two constraints, referring to the magnitude and the rate of change ϑ, should be imposed on input signals. In the case of one constraint, the input signals have the 'bang-bang' shapes, while for two constraints these signals are triangular or trapezoidal shape [7].

For one constraint, the input signal maximizing error can be determined analytically, which requires the solution of complicated integral equations. But in the case of two simultaneous constraints, determination of this error is impossible analytically.

The analytical method for determining switching instants, describing the maximizing signal with one constraint as well as relevant integral-square errors $I(x_0)$ is considered. The values of the maximum errors and corresponding number of switchings for Analog Devices and Freescale accelerometers with cut-off frequency equal to 400 Hz are listed in table. Switching instants generating the maximum value of errors and the graphical relation between the values of the error and the number of switchings are presented.

The originality of presented solutions refers to the application of the standard, presented by the ideal filter and the MathCad program used for calculation of the signal maximizing the integral-square error for one to twenty-five switchings.

2. Mathematical formulae

Let us assume that

$$y_m(t) = \int_0^t x(v)k_m(t-v)dv \tag{1}$$

$$y_s(t) = \int_0^t x(v)k_s(t-v)dv \tag{2}$$

are the outputs of the measuring system and its standard respectively, where $x(t)$ is the input signal, $k_m(t)$ and $k_s(t)$ are impulse responses for $t \in [0, T]$.

The standard presents the ideal low-pass filter of impulse response

$$k_s(t) = \frac{c}{\pi} 2\pi f_c Sinc\, [2\pi f_c(t-t_0)] \tag{3}$$

where c, t_0 and f_c denote the filter coefficient: amplification, time delay and cut-off frequency [11,12].

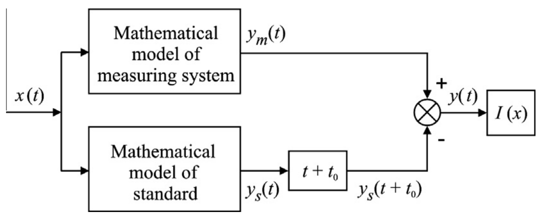

Fig. 1. Procedure for determining the integral-square error.

The input signal $x(t)$ with one constraint has the form of a 'bang-bang' with the magnitude $\pm a$. The output signal $y_s(t)$ relative to $y_m(t)$ is delayed by t_0. In order to compare these signals it is necessary to shift $y_s(t)$. Fig. 1 presents the procedure for determining the integral-square error.

The input signal $x(t)$, for n switching instants is described by the vector of the times $(t_1, t_2, \ldots, t_n)$.

On the basis of the convolution integral properties, taking into account time shifting t_0, we have

$$y_s(t+t_0) = \int_0^t x(\tau)k_s[(t+t_0)-\tau]d\tau \tag{4}$$

and

$$y(t) = y_m(t) - y_s(t) = \int_0^t x(\tau)k(t-\tau)d\tau \tag{5}$$

where

$$k(t) = k_m(t) - k_s(t+t_0) \tag{6}$$

The integral square error is defined by [10]

$$I(x) = \int_0^T y^2(t)dt = (Kx, Kx) = (K^*Kx, x), x \in X \tag{7}$$

where X is the set of signals x, K^* is conjugate of K, and

$$Kx = y(t) \tag{8}$$

On the basis of the (7) and (8) we can write

$$I(x) = (y, Kx) = (x, K^*y) \tag{9}$$

Eq. (9) can be written as follows

$$\int_0^T y(t)\int_0^t k(t-\tau)x(\tau)d\tau dt = \int_0^T x(t)[K^*y]dt \tag{10}$$

Extension the upper limit of integration on the left side of formula (10) gives

$$\int_0^T y(t)\int_0^T k(t-\tau)x(\tau)d\tau dt = \int_0^T x(t)[K^*y]dt \tag{11}$$

Changing the integration order in (11) and replacing t by τ we have

$$\int_0^T x(t)\int_0^T k(\tau-t)y(\tau)d\tau dt = \int_0^T x(t)[K^*y]dt \tag{12}$$

Because of that the integrand in (12) is equal to zero for $\tau < t$ we can write [7]

$$\int_0^T x(t)\int_t^T k(\tau-t)y(\tau)d\tau dt = \int_0^T x(t)[K^*y]dt \tag{13}$$

Taking into account (8) and (13) we have

$$K^*Kx = \int_t^T k(\tau-t)[\int_0^\tau x(v)k(\tau-v)dv]d\tau \tag{14}$$

Denoting by $x_0(t)$ the signal $x(t)$ maximizing error $I(x)$, we have

$$I(x_0) = \sup\{I(x) : x \in X\} \tag{15}$$

From (15) it result that

$$[(\partial I(x)/\partial x)|_{x_0}, x - x_0] \leqslant 0 \tag{16}$$