# A three-stage Data Envelopment Analysis model with application to banking industry

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## ARTICLE INFO

## ABSTRACT

The changing economic conditions have challenged many financial institutions to search for more efficient and effective ways to assess their operations. Data Envelopment Analysis (DEA) is a widely used mathematical programming approach for comparing the inputs and outputs of a set of homogenous Decision Making Units (DMUs) by evaluating their relative efficiency. The traditional DEA treats DMUs as black boxes and calculates their efficiencies by considering their initial inputs and their final outputs. As a result, some intermediate measures are lost in the process of changing the inputs to outputs. In this study we propose a three-stage DEA model with two independent parallel stages linking to a third final stage. We calculate the efficiency of this model by considering a series of intermediate measures and constraints. We present a case study in the banking industry to exhibit the efficacy of the procedures and demonstrate the applicability of the proposed model.

## 1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric performance evaluation method that was originally developed by Charnes et al. [19] and later extended by Banker et al. [8] to include variable returns to scale. DEA generalizes the Farrell's [36] single-input single-output technical efficiency measure to the multiple-input multiple-output

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case to evaluate the relative efficiency of peer units with respect to multiple performance measures [18,26]. The units under evaluation in DEA are called Decision Making Units (DMUs). A DMU is considered efficient when no other DMU can produce more outputs using an equal or lesser amount of inputs. The DEA generalizes the usual efficiency measurement from a single-input single-output ratio to a multiple-input multiple-output ratio by using a ratio of the weighted sum of outputs to the weighted sum of inputs [27]. Unlike parametric methods which require detailed knowledge of the process, DEA does not require an explicit functional form relating inputs and outputs (see Cooper et al. [27] and Cook and Seiford [25] for an appraisal of the theoretical foundations and developments in DEA).

Although DEA can evaluate the relative efficiency of a set of DMUs, it cannot identify the sources of inefficiency in the DMUs because conventional DEA models view DMUs as

black boxes that consume a set of inputs to produce a set of outputs [4]. In such cases, using single-stage DEA may result in inaccurate efficiency evaluation [82]. In contrast, a two-stage DEA model allows one to further investigate the structure and processes inside the DMU, to identify the misallocation of inputs among sub-DMUs and generate insights about the sources of inefficiency within the DMU [31,65].

### 1.1. Multi-stage DEA models

The existing multi-stage DEA models in the literature can be classified into two categories: closed-system and open-system models. In the closed-system DEA models, the intermediate outputs remain unchanged from one stage to another. In contrast, in the open-system DEA models, the intermediate outputs in one stage are partial inputs in a subsequent stage.

#### 1.1.1. Two-stage closed DEA system

In this type of systems, unlike the first stage, the second stage has inputs that are the intermediate variables since the outputs of the first stage are the inputs of the second stage. Fig. 1 presents a graphical representation of a closed two-stage DEA system.

Seiford and Zhu [88] used a two stage network model to measure the profitability and marketability of American commercial banks. In the first stage, they use labor and assets as inputs to produce profitability as output of the first-stage. In the second stage, they use profitability from the first stage and marketability as inputs in the second stage to produce market value and earnings per share as outputs of the second-stage. Zhu [99] also used this two-stage network for Fortune Global 500 companies. Chilingerian and Sherman [23] used a two-stage procedure to measure the physician care. This two-stage procedure has also been used to evaluate the performance of mental health care programs [86], the education sector [69], information technology [21,22], and purchasing and supply management [84].

These methods produce three separate efficiency measures for the first stage, second stage, and the DMU as a whole with no consideration of the interactions between these components. Kao and Hwang [59] showed that the performance of the DMU is a combination of the performance of two stages with a chain relation between them. The efficiencies estimated from this two-stage DEA approach was more meaningful than those estimated from the independent two-staged DEA approaches. Kao and Hwang [59] used this method in a Taiwanese insurance company and compared their results with the results from the independent stage performance measurement models.

Chen et al. [20] also proposed a DEA model similar to Kao and Hwang's [59] two-stage model, but in additive format.

#### 1.1.2. Two-stage open DEA system

In this type of system, unlike for the first stage, the second stage has other inputs in addition to the intermediate variables and the outputs of the first stage are not necessarily inputs of the second stage. Fig. 2 presents a graphical representation of an open two-stage DEA system.

There are many cases in the real-world systems in which some outputs of one stage (e.g., parts in an automobile manufacturing plant) might be delivered to customers and the rest of the output will proceed to the next stages in the manufacturing process. Most of these models are in the form of DEA network systems. In the open-system models, each stage operates as an open system and gets the inputs from outside just as it may get some from the previous stages. Golany et al. [45] designed a performance measurement system that comprised of two linked sub-systems. Each sub-system uses separate resources and produces outputs. These resources could be labor or capital. Their network DEA could calculate the performance in each sub-system as well the overall performance in the entire system.

There are a number of examples for these chained processes where each sub-process uses other resources than the outputs of the previous stage. For example, consider the *production* and *delivery* sub-systems in a manufacturing system. Labor and raw materials are the inputs in the production sub-process and the finished goods are the outputs of this sub-process. The finished goods are also considered as inputs of the delivery sub-process. Other inputs of the delivery sub-process could be drivers and trucks and the final delivered product could be the output of the delivery sub-process. Liang et al. [66] applied this network concept to the performance measurement in a supply chain using Stackelberg game strategy (or leader–follower). In their two-stage model, the second stage receives inputs other than outputs of the first stage. Another network DEA was created for systems with more than two processes based on this assumption. Castelli et al. [14] studied two-stage and two-layer DMUs. Other examples of open-system multi-stage DEA models include Färe Fare and Whirraker [35], Färe and Grosskopf [30] and Tone and Tsutsui [94,95].

### 1.2. DEA models with undesirable variables

Modeling and consideration of undesirable outputs in productivity and performance measurement date back to
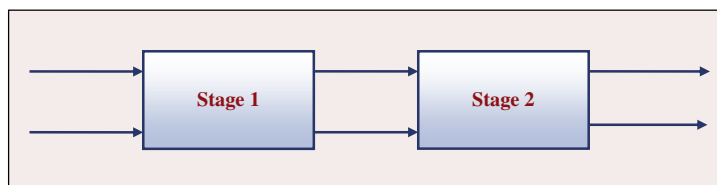


**Fig. 1.** A closed two-stage DEA system.