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A new time-domain method for frequency measurement of sinusoidal signals in critical noise conditions

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ABSTRACT

Frequency measurement of sinusoidal signals corrupted by noise is dealt with. A new time-domain, digital signal processing method, based on an enhanced version of the zero-crossing technique, is in particular proposed. To obtain the desired frequency value, the method exploits the histogram of time intervals between consecutive zero-crossing events, and works with success in the presence of zero-mean additive noise.

Several tests conducted on simulated and actual sinusoidal signals highlight the good performance of the method also in critical noise conditions, which make high-end measurement instruments like digital counters poorly fail. Moreover, the obtained results concur with those granted by competitive digital signal processing proposals operating in the frequency domain. With respect to these latter, the main advantage of the method relies on the limited computational resources required, which suggests its implementation even in low-performance processing devices for the realization of cost-effective measurement instruments.

1. Introduction

Measuring the frequency of sinusoidal signals proves to be one of the most important steps in performance assessment, maintenance and troubleshooting procedures of different systems and apparatuses, from power grids to radiofrequency transmitters, from radar devices to electromagnetic compatibility equipment. Moreover, frequency measurements (or, correspondingly, period and time interval measurements) are among the most accurate ones: instruments based on cesium oscillators, as an example, allow uncertainties in the order of a few femtoseconds to be reached [1].

Due to the high stability of their internal time reference, digital counters are widely used for direct, time-domain frequency measurements of sinusoidal signals, in ordinary laboratory and/or industrial applications [2–4]. Their resolution, however, depends both on the gate time and input signal frequency [4], and their performance strongly degrades in the presence of high additive noise levels (namely, higher than the hysteresis threshold), despite of a low-pass filter available in the input channel to cut high frequency noise components off. As for instruments for direct measurements in the frequency domain, such as spectrum analyzers, their performance is directly related to the available resolution bandwidth of the detection filter and usually not suitable for low frequency signals [8].

Concerning indirect measurements through digital signal processing methods, a number of measurement algorithms are available in the literature, operating either in a monodimensional (time or frequency) or multidimensional domain. Some of them apply zero-crossing techniques to the digitized signal [5–7], others of them exploit different, even though sometimes more complex, approaches: Euclidean, auto-regressive, discrete Fourier

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transform (DFT)-based, eigenanalysis, windowing, interpolation of spectral samples, chirplet transform (CT)-based, warblet transform (WT)-based, power spectrum estimation [9–21].

Some of these methods, namely all those in the time domain, suffer from the presence of additive noise that significantly worsens their performance [16,22–26]. That's when methods operating in different domains come into play, even though time-consuming, high-computational burden proposals, using a time-domain interpolation of sinusoidal or periodic signals, have recently been developed [27].

With special regard to frequency-domain methods, the desired frequency of the noise-free signal is in principle obtained as the value peculiar to the bin showing the maximum level in the amplitude spectrum, which is obtained through the application of the discrete Fourier transform (DFT) to the digitized signal. Major sources of uncertainty can be found in the achieved frequency resolution [28], depending both on the adopted sample rate f_s and the finite number N of acquired samples, as well as in the unwanted spectral leakage, caused by incoherent sampling in the acquisition window of finite duration. To mitigate the contribution due to the frequency resolution, some proposals exploiting spectrum interpolation techniques (as an example, two-point interpolated DFT (IpDFT) [29]) have been presented. Concerning spectral leakage effects, they have been recently tackled through high-order interpolations of the amplitude spectrum around its maximum (as an example, multi-point IpDFT [30] or weighted multi-point IpDFT (WMIpDFT) [31]) or by best fitting the theoretical spectrum of a rectangular windowed single-tone signal on the spectrum of the analyzed signal [32]. Unfortunately, all the considered proposals involve further and not negligible computational burden, thus making it unfeasible to implement them in cost-effective measurement devices.

Multidimensional approaches, such as those based on the chirplet or warblet transform, are in particular addressed to instantaneous frequency estimation of mono-component and multicomponent signals. Good results are assured in the presence of linear and nonlinear frequency trajectories versus time; no competitive outcomes are instead provided when constant frequency trajectories are involved [18,19].

A new digital signal processing method for time-domain frequency measurements of sinusoidal signals corrupted by noise is presented hereinafter. It exploits the histogram of time intervals between consecutive zero-crossing events, thus showing itself as an enhanced version of the traditional zero-crossing technique. Besides providing as accurate results as those assured by competitive solutions, the method proves to be very efficient from a computational point of view, as it only requires the counting of the number of samples or clock ticks between zero-crossing events, thus being eligible for implementation even in cost-effective measurement instruments based on low-performance processing devices, like micro-controllers featuring either an analog-to-digital converter with moderate sampling frequency or a suitable peripheral for zero-crossing detection [30].

The paper is organized as follows. Section 2 gives all theoretical background concerning the enhancement of the traditional zero-crossing technique. In Section 3, the proposed method is described in detail, and Sections 4 and 5 shows the results obtained in a number of tests conducted on simulated and actual noisy sinusoidal signals, along with their analysis through statistical quality control techniques [33–36]. Conclusive remarks are finally drawn.

2. Theoretical background

Let us suppose to measure the frequency f_0 of a sinusoidal signal $x(t)$:

$$x(t) = A\sin(2\pi f_0 t + \varphi) \tag{1}$$

when additive noise $n(t)$ is present, so that the whole signal under analysis, $y(t)$, can be modelled as:

$$y(t) = x(t) + n(t) \tag{2}$$

where A, f_0, and φ are the amplitude, frequency and initial phase of $x(t)$, respectively. Noise is considered to be stationary and wideband, with zero mean normal or uniform distribution.

The zero-crossing technique measures the desired frequency f_0 as the number of times $y(t)$ crosses the "zero" level, chosen as reference, with the same slope. In the ideal case, e.g. when no noise is added to the signal, the time between two consecutive zero-crossings is obviously equal to half the period of $x(t)$, so that:

$$f_0 = \frac{1}{T_0} = \frac{1}{2T_h} = \frac{1}{2NT_C} \tag{3}$$

where T_0 is the period of $x(t)$, T_h is the corresponding semi-period, T_c is the period of the reference time base (i.e. clock), and N represents the integer number of clock events counted during T_h.

In the presence of a noise-free, digitized sinusoidal signal (Fig. 1), an estimate of the semi-period can be obtained by measuring the time interval between the samples that occur at or immediately after two consecutive zero-crossing events. If N_{AB} is the number of samples acquired between t_A and t_B, i.e. the sampling instants occurring at or

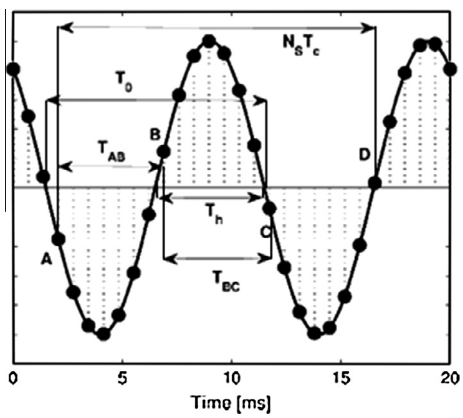

Fig. 1. Noise-free, digitized sinusoidal signal. T_0 is the actual period, T_h the corresponding semi-period, T_{AB} and T_{BC} two estimates of the semi-period.