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A fast multi-baseline and multi-frequency band phase-unwrapping algorithm

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ABSTRACT

Multi-baseline and multi-frequency band can improve the performance of phase unwrapping. This paper, taking the multi-baseline InSAR (interferometric synthetic aperture radar) system as an example, proposes a fast method for multiple-baseline and multi-frequency band phase unwrapping in the frequency domain. The basic idea is to perform a least-squares minimization of the differences between the estimated and absolute phase gradients on all interferograms simultaneously, which are all in the frequency domain. Results on real and simulated data show that the frequency-domain method yields results similar to those of the time-domain method while improving efficiency by avoiding mirroring operations.

1. Introduction

Phase unwrapping is usually defined as the reconstruction of the real value of the phase from the principal value. In addition to the application of radar interferometry [1], phase unwrapping is also applied significantly in the adaptive optics (AO) [2], nuclear magnetic resonance (NMR) [3], interferometric synthetic aperture sonar (InSAS) [4], electronic speckle pattern interferometer (ESPI) [5], seismic processing [6] and so on. Multiple-baseline InSAR technology had been widely used for DEM generation. With other parameters held constant, longer baselines enable more accurate height estimation (and better height-to-phase noise ratios), but also increase the possibility that steep terrain may cause phase aliasing, thereby degrading height measurements. On the other hand, short baselines reduce the possibility of phase aliasing, but result in poorer phase-to-height sensitivity [7]. The multi-frequency band InSAR also has a similar law. Therefore, use of multiple baselines or multi-frequency band can improve the performance of phase unwrapping. In addition, the use of two acquisitions with similar baseline lengths has the additional advantage of improving relative height accuracy [8]. At present, methods commonly used for multiple-baseline or multi-frequency band phase unwrapping include least-squares methods in the time domain [9], Kalman filtering methods [10], maximum-likelihood methods [11], Bayesian approaches [12], the Joint Subspace Projection method [13], the Minimum Cost Flow method [14,15], General formulation method [16], and others.

The least-squares method in the time domain, a robust technique, calculates the least sum of squares of the differences between the estimated and weighted sums for each baseline or each band phase gradient. In general, there are two solutions of the time-domain least-squares

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two-dimensional phase-unwrapping problem: the cosine transform [17] and the Fourier transform [18]. Only the Fourier-transform method requires mirroring operations, which are used to avoid formulation of boundary conditions and the need for a cosine transform. Like any method, the simple least-squares method has its particular defects; for instance, it cannot effectively remove systematic errors such as those arising from undersampling of the topography. Therefore, two-dimensional weighted least-squares methods have been proposed (for example, by [18]) to provide a partial solution.

In this paper, a fast phase-unwrapping algorithm is proposed for multiple-baseline and multi-frequency band interferometry in the frequency domain. Similarly to most other multiple-baseline and multi-frequency band approaches, it is an extension of the single-baseline case described in [19]. Because it avoids the mirroring operations of the time-domain algorithm [9,18], it achieves better efficiency.

2. Problem formulation

Assume that there are L interferograms and that the wrapped interferometric phase value corresponding to the kth baseline or band is $\varphi^k_{m,n}$, where $k = 1, 2, \ldots, L$, $m = 0, 1, \ldots, M-1$, $n = 0, 1, \ldots, N-1$, and M, N are the number of points in the interferogram in the range and azimuth directions respectively. Let us consider as a reference the first interferometric SAR data pair and let α_k be:

$$\alpha_k = b_1/b_k (k = 1, 2, \ldots, L) \tag{1}$$

where b_k is the normal baseline or wavelength corresponding to the kth interferometric phase diagram. Define the two directional gradient functions of the phase diagram as:

$$\begin{cases} \Delta^{k,x}_{m,n} = W\left\{\varphi^k_{m+1,n} - \varphi^k_{m,n}\right\}, & m = 0, 1, \ldots, M-2;\ n = 0, 1, \ldots, N-1 \\ \Delta^{k,y}_{m,n} = W\left\{\varphi^k_{m,n+1} - \varphi^k_{m,n}\right\}, & m = 0, 1, \ldots, M-1;\ n = 0, 1, \ldots, N-2 \end{cases} \tag{2}$$

where $W\{\cdot\}$ is a phase-wrapping function that wraps all values of its argument into the range $(-\pi, \pi)$ by adding or subtracting an integral number of 2π radians from its argument, x and y are the range and azimuth directions respectively of the interferogram. Let us extend the definition of $\Delta^{k,x}_{m,n}$ and $\Delta^{k,y}_{m,n}$ to the grid of points (m, n), for $m = 0, 1, \ldots, M-1$; $n = 0, 1, \ldots, N-1$ by defining:

c

$$\begin{cases} \Delta^{k,x}_{M-1,0} = c, \Delta^{k,y}_{0,N-1} = c & \forall c \in (-\pi, \pi) \\ \Delta^{k,x}_{M-1,n} = \Delta^{k,x}_{M-1,n-1} + \Delta^{k,y}_{1,n-1} - \Delta^{k,y}_{M-1,n-1}, & n = 1, \ldots, N-1 \\ \Delta^{k,y}_{m,N-1} = \Delta^{k,y}_{m-1,N-1} + \Delta^{k,x}_{m-1,1} - \Delta^{k,x}_{m-1,N-1}, & m = 1, \ldots, M-1 \end{cases} \tag{3}$$

The extension can satisfy the irrotational-field boundary condition after further periodic expansion by discrete Fourier transform [19].

Assuming that $F^{k,x}_{p,q}$ and $F^{k,y}_{p,q}$ are two-dimensional fast Fourier transforms (2D FFTs) of $\Delta^{k,x}_{m,n}$ and $\Delta^{k,y}_{m,n}$ respectively, the irrotational-field condition [19] in the frequency domain can be expressed as follows:

$$F^{k,x}_{p,q}(e^{2\pi i \frac{q}{N}} - 1) = F^{k,y}_{p,q}(e^{2\pi i \frac{p}{M}} - 1) \quad p = 0, 1, \ldots, M-1; \quad q = 0, 1, \ldots, N-1 \tag{4}$$

Let $\phi_{m,n}$ be the least-squares solution of a set of multiple-baseline or multi-frequency band phase-unwrapping equations in the frequency domain and $\tilde{\Delta}^x_{m,n}$, $\tilde{\Delta}^y_{m,n}$ its two directional gradient functions. $\tilde{F}^x_{p,q}$ and $\tilde{F}^y_{p,q}$ are the respective 2D FFTs. In a rotational field, Eq. (4) does not hold. Suppose that $\tilde{F}^x_{p,q}$ and $\tilde{F}^y_{p,q}$ satisfy the irrotational-field condition:

$$\tilde{F}^x_{p,q}(e^{2\pi i \frac{q}{N}} - 1) = \tilde{F}^y_{p,q}(e^{2\pi i \frac{p}{M}} - 1) \quad p = 0, 1, \ldots, M-1; \quad q = 0, 1, \ldots, N-1 \tag{5}$$

The goal of the least-squares algorithm in the frequency domain is to minimize the functional:

$$J = \sum_{k=1}^{L}\left[\sum_{p=0}^{M-2}\sum_{q=0}^{N-1}\left|\alpha_k F^{k,x}_{p,q} - \tilde{F}^x_{p,q}\right|^2 + \sum_{p=0}^{M-1}\sum_{q=0}^{N-2}\left|\alpha_k F^{k,y}_{p,q} - \tilde{F}^y_{p,q}\right|^2\right] \to \min \tag{6}$$

where

$$\sum_{k=1}^{L}\left[\left|\alpha_k F^{k,x}_{p,q} - \tilde{F}^x_{p,q}\right|^2\right] = L\left[\tilde{F}^x_{p,q} - \frac{1}{L}\sum_{k=1}^{L}\alpha_k F^{k,x}_{p,q}\right]^2 + \frac{1}{L}\sum_{k=1}^{L}(\alpha_k F^{k,x}_{p,q})^2 - \left(\frac{1}{L}\sum_{k=1}^{L}\alpha_k F^{k,x}_{p,q}\right)^2 \tag{7}$$

$$\sum_{k=1}^{L}\left[\left|\alpha_k F^{k,y}_{p,q} - \tilde{F}^y_{p,q}\right|^2\right] = L\left[\tilde{F}^y_{p,q} - \frac{1}{L}\sum_{k=1}^{L}\alpha_k F^{k,y}_{p,q}\right]^2 + \frac{1}{L}\sum_{k=1}^{L}(\alpha_k F^{k,y}_{p,q})^2 - \left(\frac{1}{L}\sum_{k=1}^{L}\alpha_k F^{k,y}_{p,q}\right)^2 \tag{8}$$

Therefore,

$$J = L\left\{\sum_{p=0}^{M-2}\sum_{q=0}^{N-1}[\tilde{F}^x_{p,q} - \frac{1}{L}\sum_{k=1}^{L}\alpha_k F^{k,x}_{p,q}]^2 + \sum_{p=0}^{M-1}\sum_{q=0}^{N-2}[\tilde{F}^y_{p,q} - \frac{1}{L}\sum_{k=1}^{L}\alpha_k F^{k,y}_{p,q}]^2\right\} + C \to \min \tag{9}$$

where $C = \frac{1}{L}\sum_{k=1}^{L}(\alpha_k F^{k,x}_{p,q})^2 - (\frac{1}{L}\sum_{k=1}^{L}\alpha_k F^{k,x}_{p,q})^2 + \frac{1}{L}\sum_{k=1}^{L}(\alpha_k F^{k,y}_{p,q})^2 - (\frac{1}{L}\sum_{k=1}^{L}\alpha_k F^{k,y}_{p,q})^2$ is constant.

Therefore, Eq. (9) is equivalent to:

$$J' = \sum_{p=0}^{M-2}\sum_{q=0}^{N-1}\left[\tilde{F}^x_{p,q} - \hat{F}^x_{p,q}\right]^2 + \sum_{p=0}^{M-1}\sum_{q=0}^{N-2}\left[\tilde{F}^y_{p,q} - \hat{F}^y_{p,q}\right]^2 \to \min \tag{10}$$

where $\hat{F}^x_{p,q} = \frac{1}{L}\sum_{k=1}^{L}\alpha_k F^{k,x}_{p,q}$ and $\hat{F}^y_{p,q} = \frac{1}{L}\sum_{k=1}^{L}\alpha_k F^{k,y}_{p,q}$.

Let $C_1 = (e^{2\pi i \frac{p}{M}} - 1)$, $C_2 = (e^{2\pi i \frac{q}{N}} - 1)$. Simultaneous solution of Eqs. (5) and (10) yields:

$$\tilde{F}^x_{p,q} = \frac{C_1\overline{C}_1\hat{F}^x_{p,q} + C_1\overline{C}_2\hat{F}^y_{p,q}}{C_1\overline{C}_1 + C_2\overline{C}_2} \tag{11}$$

$$\tilde{F}^y_{p,q} = \frac{C_2\overline{C}_1\hat{F}^x_{p,q} + C_2\overline{C}_2\hat{F}^y_{p,q}}{C_1\overline{C}_1 + C_2\overline{C}_2} \tag{12}$$

where $\overline{C}_1$, $\overline{C}_2$ are the conjugates of C_1, C_2.