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Cognitive mechanisms underlying third graders' arithmetic skills: Expanding the pathways to mathematics model

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ABSTRACT

A modified pathways to mathematics model was used to examine the cognitive mechanisms underlying arithmetic skills in third graders. A total of 269 children were assessed on tasks tapping the four pathways and arithmetic skills. A path analysis showed that symbolic number processing was directly supported by the linguistic and approximate quantitative pathways. The direct contribution from the four pathways to arithmetic proficiency varied; the linguistic pathway supported single-digit arithmetic and word problem solving, whereas the approximate quantitative pathway supported only multi-digit calculation. The spatial processing and verbal working memory pathways supported only arithmetic word problem solving. The notion of hierarchical levels of arithmetic was supported by the results, and the different levels were supported by different constellations of pathways. However, the strongest support to the hierarchical levels of arithmetic were provided by the proximal arithmetic skills.

Introduction

Sufficient mathematical skills is a prerequisite to adequately function in society (Butterworth, Varma, & Laurillard, 2011). Prior research shows that basic arithmetic skills are precursors to neces-

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sary mathematical skills (Duncan et al., 2007; Hooper, Roberts, Sideris, Burchinal, & Zeisel, 2010). However, children's arithmetic skills vary already at an early age (Anders, Grosse, Rossbach, Ebert, & Weinert, 2013; Berch & Mazzocco, 2007; Duncan et al., 2007). How can these large individual differences be accounted for? One line of research has focused on mapping the cognitive mechanisms underlying children's basic mathematical skills. It provides evidence that symbolic number processing skills account for individual differences in children's arithmetic skills above and beyond general cognitive abilities (Fuchs et al., 2010a; Geary, 2011; Martin, Cirino, Sharp, & Barnes, 2014; Träff, 2013). General cognitive abilities have also been found to provide independent contributions to children's arithmetic achievement (Fuchs et al., 2010b; Geary, 2011; Martin et al., 2014; Passolunghi & Lanfranchi, 2012; Östergren & Träff, 2013).

The current study expands this line of research by employing an extended version of the *pathways to mathematics model* developed by LeFevre et al. (2010).

The pathways to mathematics model

The original model includes three cognitive pathways (quantitative, linguistic, and spatial attention) underlying the development of early numeracy skills (i.e., before formal instruction) and children's acquisition of mathematical knowledge in school. The quantitative pathway is consistent with the assumption of a core number system, providing the foundation for the language-based symbolic number system (Butterworth, 1999; Feigenson, Dehaene, & Spelke, 2004; Gelman & Butterworth, 2005; Piazza, 2010). The linguistic pathway plays an important role in the development of the language-based symbolic number system when children learn to speak (LeFevre et al., 2010). Children first learn the number words by rote (i.e., rote counting skill) and then establish a verbal number code by mapping the number words onto the core number system (Dehaene, 1992; Geary, 2013; von Aster & Shalev, 2007). Thereafter, the Arabic numerals are learned and connected to the core number system, where they integrate with the previously learned verbal number code (Geary, 2013; Krajewski & Schneider, 2009; von Aster & Shalev, 2007). The verbal number code is used for counting and enumeration (Dehaene, 1992; Logie & Baddeley, 1987) and is assumed to subserve the solution of single-digit arithmetic problems through counting strategies and establishing and retrieving number facts (Dehaene, 1992; Geary, 1993).

The importance of the spatial attention pathway to early number and mathematical skills is founded on research showing that visual–spatial working memory contributes to young children's mathematical performance (Andersson & Lyxell, 2007; Holmes, Adams, & Hamilton, 2008; Krajewski & Schneider, 2009; McKenzie, Bull, & Gray, 2003). Based on their model, LeFevre et al. (2010) proposed two hypotheses. First, all three pathways contribute independently to basic symbolic number processing skills. Second, the pathways' unique and relative contributions to formal mathematics vary depending on the complexity and demand of the tasks.

Model-based studies

Both hypotheses received strong support from LeFevre et al.'s (2010) original study performed on 4- to 6-year-olds. The spatial attention pathway (visual–spatial working memory) predicted both measures of early numeracy knowledge—nonverbal arithmetic and number naming—as well as number line estimation but did not predict digit comparison 2 years later. The linguistic pathway (vocabulary and phonological awareness) predicted number naming, number line estimation, and digit comparison but not nonverbal arithmetic. The quantitative pathway (subitizing) was significantly related to nonverbal arithmetic, number line estimation, and digit comparison but not to number naming. Regarding the mathematical outcomes (Hypothesis 2), the spatial attention and linguistic pathways accounted for variability in calculation, geometry, and measurement, whereas the quantitative pathway predicted only calculation.

Three other studies have more or less explicitly tested the pathways model. Cirino (2011) examined 6- and 7-year-olds and used similar tasks as LeFevre et al. (2010) to tap the spatial attention pathway (visuospatial working memory) and the linguistic pathway (phonological awareness and rapid automatized naming) but used dot comparison to tap the quantitative pathway. All three pathways