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Brief Report

A set for relational reasoning: Facilitation of algebraic modeling by a fraction task

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A B S T R A C T

Recent work has identified correlations between early mastery of fractions and later math achievement, especially in algebra. However, causal connections between aspects of reasoning with fractions and improved algebra performance have yet to be established. The current study investigated whether relational reasoning with fractions facilitates subsequent algebraic reasoning using both pre-algebra students and adult college students. Participants were first given either a relational reasoning fractions task or a fraction algebra procedures control task. Then, all participants solved word problems and constructed algebraic equations in either multiplication or division format. The word problems and the equation construction tasks involved simple multiplicative comparison statements such as “There are 4 times as many students as teachers in a classroom.” Performance on the algebraic equation construction task was enhanced for participants who had previously completed the relational fractions task compared with those who completed the fraction algebra procedures task. This finding suggests that relational reasoning with fractions can establish a relational set that promotes students’ tendency to model relations using algebraic expressions.

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Introduction

Recent research has established correlational links between early success with fractions and general math achievement (Siegler et al., 2012; Torbeyns, Schneider, Xin, & Siegler, 2015) and in particular with algebra performance (Booth & Newton, 2012; Booth, Newton, & Twiss-Garrity, 2014; DeWolf, Bassok, & Holyoak, 2015b; Empson & Levi, 2011; Wu, 2001, 2009). The importance of early learning and mastery of fractions for subsequent math performance has been widely recognized, but researchers have yet to establish a more direct causal connection between specific aspects of fraction understanding and algebra performance.

Although they cannot establish causality, correlational studies have provided clues to elements of fraction understanding that are potentially important for subsequent algebra understanding. Using multiple regression analyses, DeWolf and colleagues (2015b) found that middle school algebra performance was uniquely predicted by two factors: understanding of fractions as relations and number line estimation with decimals. In contrast, although number line estimation with fractions and procedural knowledge about fractions in algebra equations were also significantly correlated with algebra performance, neither proved to be a unique predictor of algebra performance over and above relational fraction understanding and number line estimation with decimals. Thus, these findings suggest that the *relational* aspect of fraction understanding, as opposed to procedural knowledge of fractions in algebra equations or facility with fraction magnitudes, has a particularly direct connection to success in algebra.

There are theoretical reasons to expect that relational understanding of fractions may support subsequent acquisition of algebra. Understanding algebra largely depends on grasping abstract relations between entities and numbers. After all, algebraic equations and expressions are meant to convey abstract relations. For example, Sfard and Linchevski (1994) argued that success in algebra depends on students moving beyond simple mastery of how to carry out procedures to find solutions. Beyond procedural knowledge, students must understand that algebraic expressions convey relations between quantities and that a general process may be used to find an unknown quantity. For example, if you have decided to split a restaurant bill among four people but do not yet know what the total cost of the bill is (b), you can express this relation as $b/4$ or $(1/4)b$. This expression simultaneously represents the individual cost for each of the four people and also a procedure that can be used to derive that specific cost depending on the actual amount of the bill.

Algebraic modeling

One example of an area of algebra that is particularly difficult for both children and adults involves understanding and generating algebraic expressions representing multiplicative comparisons (Clement, Lochhead, & Soloway, 1979; Fisher, Borchert, & Bassok, 2011; Martin & Bassok, 2005). For example, both children and adults have difficulty in generating the correct algebraic equation for statements such as "There are 4 times as many students as teachers in a classroom." Participants often reverse the correct order of the variables, producing $4S = T$ rather than the correct equation $4T = S$. This common error has generally been interpreted as reflecting direct translation of the components of the sentence (Fisher et al., 2011; Graf, Bassok, Hunt, & Minstrell, 2004; Herscovics, 1989; Hinsley, Hayes, & Simon, 1977; Mayer & Hegarty, 1996). Specifically, participants translate "4 times students" as $4S$, following the surface order in which the components (4, times, and students) are mentioned in the sentence instead of appreciating the underlying direction of the comparison between students and teachers.

This direct translation strategy appears to be a procedure used by students who do not understand how to appropriately model the relations in the equation. In addition, this strategy may also be used as a shortcut heuristic even by students who *do* understand algebraic modeling (Graf et al., 2004). Direct translation is an effective method for constructing algebraic equations in many problems (e.g., "The number of students is 6 times the number of professors"). Thus, this strategy is not always maladaptive, and in many cases it can reduce cognitive load in solving the problem.

To distinguish genuine misconceptions about the relational structure of a problem from inappropriate use of a shortcut strategy, Fisher and colleagues (2011) gave participants an algebraic modeling