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Arithmetic word problems describing discrete quantities: E.E.G evidence for the construction of a situation model☆

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ABSTRACT

In this research, university students were asked to solve arithmetic word problems constructed either with discrete quantities, such as apples or marbles, or continuous quantities such as meters of rope or grams of sand. An analysis of their brain activity showed different alpha levels between the two types of problems with, in particular, a lower alpha power in the parieto-occipital area for problems describing discrete quantities. This suggests that processing discrete quantities during problem solving prompts more mental imagery than processing continuous quantities. These results are difficult to reconcile with the schema theory, according to which arithmetic problem solving depends on the activation of ready-made mental frames stored in long-term memory and triggered by the mathematical expression used in the texts. Within the schema framework, the nature of the objects described in the text should be quickly abstracted during problem solving because it cannot impact the semantic structure of the problem. On the contrary, our results support the situation model theory, which places greater emphasis on the problem context in order to account for individuals' behaviour. On a more methodological point of view, this study constitutes the first attempt to infer the characteristics of individual's mental representations of arithmetic text problems from EEG recordings. This opens the door for the application of brain activity measures in the field of arithmetic word problem.

1. Introduction

As testified by several international surveys, arithmetic word problem solving is an area of mathematics in which student performance is particularly low (e.g., Mullis, Martin, Foy, & Arora, 2012; OCDE, 2014). One of the main explanations of this poor performance is the difficulty that children encounter in the construction of an adequate mental representation from the text of the problem (Coquin-Viennot & Moreau, 2003; De Corte & Verschaffel, 1985; De Corte, Verschaffel, & De Win, 1985; Thevenot, 2008; Thevenot, Devidal, Barrouillet, & Fayol, 2007; Verschaffel, Greer, & De Corte, 2000). Therefore, examination of the relevance of different theories about the nature of the internal representation derived from a text problem is crucial to better understand and overcome children's difficulties.

Two main theories have been developed in order to account for arithmetic word problem solving. On the one hand, the schema theory states that word problems are solved through the activation of abstract frames from long-term memory (Kintsch & Greeno, 1985; Riley & Greeno, 1988; Riley, Greeno, & Heller, 1983). Those frames, or schemas, are triggered by the relational words used in the text of the problem, such as the mathematical expressions "more than", "less than" or "altogether". Once a schema is activated, the variables described in the text are entered into the empty slots of the frame until the problem is completely specified. At this point, the arithmetical procedure linked to the schema can be initiated and the calculations can be performed. On the other hand, the situation model theory hypothesizes that a non-mathematical representation is constructed before a more formal mathematical representation (Brissiaud & Sander, 2010; Nathan, Kintsch, & Young, 1992; Reusser, 1989; Staub & Reusser, 1995; Thevenot, Barrouillet, & Fayol, 2004; Thevenot & Oakhill, 2005). According to this theory, only the pieces of knowledge that are relevant for the understanding of the situation described in the problem are mobilized from long-term memory. The situation model is therefore mainly constructed ad-hoc in working-memory. Within this framework, the representation is constructed each time a problem is encountered, and its organization depends on specific contextual elements, which, in

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contrast, play very little role within the schema theory.

Evidence that the schema theory cannot always account for individuals' behaviour has been provided by Thevenot (2010) who demonstrated that, contrary to Kintsch and Greeno's assumption (1985), the mathematical expression used in a text of a problem does not play a key role in the solution process. In the experiment, the problem solving phase was followed by an unexpected recognition task of the problem questions. The results showed that the solvers were often confused when the mathematical expression used in the problem presented to them differed from the expression used in the original problem but depicted the same relation. For example, when the original problem was associated with the question "How many marbles do John and Paul have more than Bill?", participants often erroneously thought that the problem with the question "How many marbles does Bill have fewer than John and Paul?" was initially presented. This finding supports the view that only the relation between the elements described in a problem is mentally represented by individuals and not the specific mathematical expression used in the text. Again, these results are at odds with the schema theory, which considers the mathematical expression to be at the heart of the mental representation. On the contrary, they fit perfectly in a situation model framework according to which problem solvers construct an integrated mental representation of the situation described in the text (Bransford, Barclay, & Franks, 1972; Yuill & Oakhill, 1991). Within this framework, it is easily understandable that solvers are more likely to remember the gist of the problem than its literal formulation (Mani & Johnson-Laird, 1982; Zwaan & Radvansky, 1998).

In the current paper, we aim to test further the relevance of the schema and the situation model theories within the domain of arithmetic word problems. To this purpose, we manipulated the nature of the objects used in the text and reasoned that, if a situation model is constructed by individuals, it should influence the solution process. More precisely, we expect that continuous quantities (e.g., kilos of sand) and discrete quantities (e.g., number of apples) will not be mentally represented by individuals in the same way. Discrete quantities are countable and easily imageable (e.g., 3 apples) but continuous quantities are not associated with such precise representations and are therefore necessarily more difficult to visualize (e.g., 3 kilos of sand). Nevertheless, the schema theory would not predict differences in processing depending on the nature of the object described in the text because it is not supposed to affect the type of schema that will be triggered, which mainly depends on the mathematical expression used in the text. Therefore, the nature of the objects associated with quantities should quickly be abstracted during reading because it is not relevant for the solution task.

The level of mental imagery that word problems elicit seems difficult to investigate through behavioural studies. However, recording brain activity in adults during problem solving could be especially appropriate in order to examine our hypotheses. Therefore, we studied the level of mental imagery prompted by the nature of the objects described in the problem texts using electrophysiological responses. This is possible through the observation of different types of neural oscillations and more particularly of the alpha rhythm. The alpha band with a frequency going from 8 to 13 Hz was first thought to reflect cortical idling because it is recordable during the resting state (Pfurtscheller, Stancák Jr., & Neuper, 1996). Nowadays, electro-activity within the alpha band is more precisely considered as reflecting inhibition of the brain area in which it is recorded (Jensen & Mazaheri, 2010; Laufs et al., 2003; Toscani, Marzi, Righi, Viggiano, & Baldassi, 2010). Therefore and stated differently, when alpha activity is low, it is considered to reflect individuals' engagement in a cognitive activity associated with the recorded brain area. Accordingly, a decrease of the alpha power has been recorded when individuals engage in mental imagery (Barrett & Ehrlichman, 1982; Bértolo, 2005; Gualberto Cremades, 2002), especially in the parieto-occipital region, which is particularly involved in visual mental imagery (D'Esposito et al., 1997; Knauff, Kassubek, Mulack, & Greenlee, 2000; Kosslyn et al., 1999; Salenius, Kajola, Thompson, Kosslyn, & Hari, 1995; Short, 1953; Williamson, Kaufman, Lu, Wang, & Karron, 1997).

Consequently, if a situation model is constructed during problem solving, the level of imagery, and therefore the level of alpha activity, will vary with the nature of the objects described in the text of the problem. However, if problem solving consists in a more mechanistic activation of a schema from long-term memory, such a difference is not expected. In order to test these predictions, we asked adult participants to solve arithmetic word problems constructed either with discrete (e.g. apples) or continuous quantities (e.g. kilos of sand). Each problem describing discrete quantities was matched with a problem describing continuous quantities. Other variables, such as the number of words in the problems, the number of syllables and the frequency of the nouns associated with quantities were kept constant.

2. Method

2.1. Participants

Eighteen undergraduate students from the University of Geneva took part in the experiment for course credits. They were all native French speakers and they were aged between 18 and 29 (mean: 19.78 years, standard deviation: 2.49 years), three of them were men and three of them were left-handed.

2.2. Materials and procedure

Participants were asked to solve Compare problems involving static situations in which the unknown quantity could correspond to the difference between two sets (e.g., Joe has 6 apples. Tom has 4 apples. How many apples does Joe have more than Tom?), the compared set (e.g., Joe has 6 apples. Tom has 2 more apples than Joe. How many apples does Tom have?), or the reference set (e.g., Joe has 6 apples. He has 2 more apples than Tom. How many apples does Tom have?) (Riley et al., 1983). Twelve problems were constructed using discrete quantities such as *peaches* or *cars* and twelve problems were constructed using continuous quantities such as *kilos of sand* or *meters of rope*. Problems with discrete and continuous quantities were matched on the numbers of words contained in the text. Moreover, for a given problem, the nouns corresponding to the objects described in the text were matched on the number of letters, the number of syllables, the number of orthographic neighbours and had similar frequency ratio with a minimum of 28 occurrences per million (based on Frantext Corpus used in Lexique 2; New, Pallier, Brysbaert, & Ferrand, 2004). Moreover, the level of familiarity of each expression was assessed by 140 undergraduate students and an item-based analysis showed that it did not differ depending on quantity types (i.e, discrete or continuous), $F < 1$. Finally, the numbers used in corresponding problems (i.e. continuous vs. discrete) were the same (see Table 1 for an example). They comprised the numbers between 1 and 9, and the answer to the problems, which was reached either after a simple addition or subtraction, did not exceed 10.

Each problem was presented 4 times with different numbers across 4 experimental blocks. Therefore, each participant solved 96 problems: 4 (blocks) × 12 (problems) × 2 (quantity type: discrete or continuous). The order of presentation of the 24 problems (i.e., 12 with discrete and 12 with continuous quantities) was completely randomized in each block and a different randomization was created for each participant. Before the experiment, participants were presented with four warm-up trials.

The experiment was presented using E-Prime software. After a fixation cross lasting 500 ms, the text of the problem without the question was displayed on the screen. The question appeared after the participant pressed the enter key. As we will see in the next paragraph, this segmented presentation of the text allowed the isolation of a