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Effects of non-symbolic arithmetic training on symbolic arithmetic and the approximate number system

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A B S T R A C T

The approximate number system (ANS) is an innate cognitive template that allows for the mental representation of approximate magnitude, and has been controversially linked to symbolic number knowledge and math ability. A series of recent studies found that an approximate arithmetic training (AAT) task that draws upon the ANS can improve math skills, which not only supports the existence of this link, but suggests it may be causal. However, no direct transfer effects to any measure of the ANS have yet been reported, calling into question the mechanisms by which math improvements may emerge. The present study investigated the effects of a 7-day AAT and successfully replicated previously reported transfer effects to math. Furthermore, our exploratory analyses provide preliminary evidence that certain ANS-related skills may also be susceptible to training. We conclude that AAT has reproducible effects on math performance, and provide avenues for future studies to further explore underlying mechanisms - specifically, the link between improvements in math and improvements in ANS skills.

1. Introduction

The approximate number system (ANS) is a primitive cognitive system present across many species, both human and non-human alike. It endows the individual with an intuitive, albeit approximate, understanding of magnitude, and underlies such common human faculties as estimating the number of apples on a tree or the number of jelly beans in a jar. This ability is apparent even in human infants prior to the onset of any formal numerical instruction, and is thought to provide a natural template upon which to build an understanding of symbolic numbers (Lipton & Spelke, 2005; Mundy & Gilmore, 2009; Piazza, 2010).

Much behavioral evidence supports a close link between the representation of ANS numerosities and exact symbolic numbers, and suggests that the two share similar behavioral signatures. Most notably, both are susceptible to numerical distance effects such that identifying the larger of two quantities is more difficult the closer the quantities are together. For example, in a prototypical dot comparison task to measure ANS acuity, discriminating an array of 10 dots from an array of 12 is harder than discriminating 10 from 20 dots, and this distance effect can be observed in terms of both increasing reaction time as well as decreasing accuracy the smaller the ratio between the two dot arrays becomes (Dehaene, Dehaene-Lambertz, & Cohen, 1998). Similarly, a symbolic distance effect has robustly demonstrated longer reaction time latencies when identifying the larger of two closely spaced numbers such as 5 and 6, as opposed to relatively more distant numbers such as 5 and 9 (Moyer & Landauer, 1967). Additionally, controlling for the numerical distance between two quantities, a size effect also exists in that larger numbers or numerosities are more difficult (i.e., longer reaction times) to distinguish than smaller ones (Buckley & Gillman, 1974; Dehaene et al., 1998).

Taking this relationship one step further, the acuity of the ANS has also been shown to predict formal math ability (reviewed in Feigenson, Libertus, & Halberda, 2013). This relationship has been mainly explored in young children (Libertus, Feigenson, & Halberda, 2013a, 2013b; Odic et al., 2016), but exists throughout the school years (Halberda, Mazzocco, & Feigenson, 2008), and even correlates with SAT and GRE quantitative scores in adolescents and young adults (Dewind & Brannon, 2012; Libertus, Odic, & Halberda, 2012; Wang, Halberda, & Feigenson, 2017). Moreover, the link has been demonstrated in individuals with poor math ability (e.g., Mazzocco, Feigenson, & Halberda, 2011; Olsson, Ostergren, & Traff, 2016; Piazza et al., 2010), typical math ability (Feigenson et al., 2013), as well as precocious math ability (Wang et al., 2017), suggesting that the influence of the ANS on math is pervasive not only across a broad age range,

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but also across different levels of education and math proficiency. However, these findings are not without controversy, and several null reports have been published contesting the relationship between ANS and formal math, both in children as well as in adults (reviewed in Feigenson et al., 2013). The reasons for this inconsistency likely relate at least in part to psychometric differences across studies and low concurrent validity among ANS tests (Dietrich, Huber, & Nuerk, 2015; Gilmore, Attridge, & Inglis, 2011; Smets, Gebuis, Defever, & Reynvoet, 2014). Different tasks purporting to index the ANS often have low correlations with each other, and therefore, different studies may not always be measuring the same underlying construct. Nevertheless, throughout this noise, cumulative meta-analytic evidence still supports the existence of an overall small, but reliable correlation ($r = 0.20$ to 0.24) between math and ANS acuity (Chen & Li, 2014; Schneider et al., 2016), supporting the contention that the ANS is in fact related to mathematical and numerical knowledge.

The prospect of ANS plasticity is therefore of considerable interest, as it may implicate downstream effects on higher order skills. Although this effect is small, and certainly less predictive of later math performance than the more commonly studied symbolic processing of numbers (de Smedt, Noel, Gilmore, & Ansari, 2013), it still represents a heretofore largely untapped avenue for intervention. Moreover, intervention can occur at an unprecedentedly early age since the ANS is behaviorally present even in infancy (Starr, Libertus, & Brannon, 2013). From there, ANS acuity gradually increases throughout childhood (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Piazza et al., 2010) and even throughout the young adult years, not peaking until around age 30 (Halberda et al., 2012), suggesting a high degree of plasticity. Additionally, education, particularly in quantitative fields, has also been shown to lead to a more refined ANS (Castronovo & Gobel, 2012; Halberda et al., 2008; Lindskog, Winman, & Juslin, 2014; Piazza, Pica, Izard, Spelke, & Dehaene, 2013). Therefore, ANS acuity, though innate, may also be highly susceptible to experience and environmental input. In fact, targeted interventions involving repeated practice on number sense tasks have sought to test this plasticity more specifically, demonstrating improved ANS acuity in typically developing children (Odic, Hock, & Halberda, 2014), improved acuity and number processing in dyscalculic children (Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006), rapid learning effects in response to trial-by-trial feedback in healthy adults (Dewind & Brannon, 2012; Lindskog, Winman, & Juslin, 2013), and generalized magnitude discrimination improvements when coupling ANS exposure with transcranial random noise stimulation (Cappelletti et al., 2013).

Park and Brannon (2013, 2014) took this one step further and demonstrated that training to improve ANS skills via an approximate arithmetic training (AAT) task can also improve symbolic arithmetic skills among college students, as measured by addition and subtraction of Arabic numerals. Given the correlations between the ANS and mathematics performance throughout the school years, up to and including SAT and GRE scores (Dewind & Brannon, 2012; Libertus et al., 2012), this finding suggests a potential causal link between the ANS and mathematics that can be exploited by targeted training that fosters the bottom-up development of numeracy skills at a core, foundational level. This finding was later replicated among preschoolers with a standardized test battery of math achievement using a similar training intervention (Park, Bermudez, Roberts, & Brannon, 2016), and Wang, Odic, Halberda, and Feigenson (2016) also concurrently found that even brief exposure to an ANS acuity task over a single session can improve formal math ability among preschoolers if the ANS trials are presented in a scaffolded manner (i.e., easier trials first). Despite these promising initial results, however, evidence for true plasticity at the level of the ANS has been criticized and is still inconclusive (Lindskog & Winman, 2016; Lindskog et al., 2013; Szucs & Myers, 2017), casting much uncertainty on what exactly mediates the improvements observed in math. One issue is that AAT, which involves the approximate addition and subtraction of dot clouds of varying numerosity, may be training additional processes beyond the ANS itself. Though Park and Brannon (2014) ruled out secondary processes such as visual working memory, covert symbolic arithmetic practice during the AAT, or general placebo effects, they were also not able to demonstrate any convincing training-related improvements on a measure of ANS acuity, and it is still an open question as to whether math improvements after AAT are specifically related to changes in the ANS per se (e.g., see Szucs & Myers, 2017).

The present study, therefore, has two goals. First, we attempt an independent replication of the transfer effects of AAT on symbolic arithmetic proficiency. Second, we seek to systematically explore direct transfer effects of AAT to ANS-related skills. With respect to the second goal, we aim to improve on the methodology used by Park & Brannon in several ways. First, while Park and Brannon (2014) used a single measure (a nonsymbolic comparison task) to index the ANS, we use a battery of different tasks, evaluating both nonsymbolic and symbolic tests of comparison, estimation, and nonverbal counting. Evidence suggests that the ANS may not represent a unitary construct, and that different metrics do not correlate well with each other (Gilmore et al., 2011; Smets et al., 2014). Therefore, a valid assessment of training-related ANS change would likely require multiple measures. Moreover, our use of both nonsymbolic as well as symbolic versions of each of our tasks allows an evaluation of both specific and general transfer to number sense. If improvements in math are truly a result of specific improvements in the ANS, then these improvements in nonsymbolic discrimination must also be generalizable to the symbolic domain as well. Finally, we seek to maximize the chances of transfer by creating outcome measures that more closely mirror the relevant characteristics of the training regimen. For example, one issue with the nonsymbolic comparison task used in Park and Brannon (2014) to assess near transfer to the ANS is that it involved the comparison of ratios typically much smaller than what was trained. Fig. 2 of Park and Brannon (2014) shows a log difference level of just over 0.5 at the end of six training sessions, which corresponds to discriminating dot arrays that are separated by approximately a ratio of 1.5 to 1. However, their nonsymbolic comparison task tested participants on ratios that were almost all below 1.25 to 1, a range on which they received very little training. Therefore, the tasks used in the present study, including our version of the nonsymbolic comparison task, incorporate magnitude information designed to contain greater overlap with the trained numerosities, and our statistical analyses are designed to investigate the degree to which this matters by systematically evaluating group differences across different magnitude ranges.

Another issue is that Park & Brannon controlled for continuous perceptual cues such as average dot size and total surface area in their transfer task, but not in the training task. Though such non-numerical stimulus control has recently become common practice in the literature (c.f., Dietrich et al., 2015), and is arguably a more pure way to measure the abstraction of numerical information, unconfounded by other continuous perceptual cues, this makes the task much harder for participants (Agrillo, Piffer, & Bisazza, 2011; Dietrich et al., 2015; Gebuis & Reynvoet, 2012b), and may not entirely engage the same cognitive processes that were trained considering that the training task did not control for such perceptual cues. In order to maximize chances of detecting transfer effects, it is important to increase process overlap with the trained task (c.f., Jaeggi et al., 2010; Loosli, Buschkuehl, Perrig, & Jaeggi, 2012; Lustig, Shah, Seidler, & Reuter-Lorenz, 2009). Therefore, this required making a design choice on our part to either control for non-numerical cues in our training task, or to keep the training task as is and remove such controls from the transfer tasks. We opted for the latter choice in order to keep the training as consistent as possible to that of Park and Brannon (2013, 2014), reasoning that this approach would maximize chances of replicating the transfer effects to symbolic arithmetic proficiency, as any attempt to evaluate the underlying mechanisms of training would otherwise be moot.

Moreover, it has been demonstrated that both humans and non-