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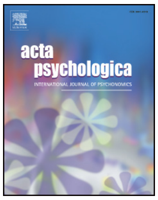

Approximate number and approximate time discrimination each correlate with school math abilities in young children

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ABSTRACT

What is the relationship between our intuitive sense of number (e.g., when estimating how many marbles are in a jar), and our intuitive sense of other quantities, including time (e.g., when estimating how long it has been since we last ate breakfast)? Recent work in cognitive, developmental, comparative psychology, and computational neuroscience has suggested that our representations of approximate number, time, and spatial extent are fundamentally linked and constitute a “generalized magnitude system”. But, the shared behavioral and neural signatures between number, time, and space may alternatively be due to similar encoding and decision-making processes, rather than due to shared domain-general representations. In this study, we investigate the relationship between approximate number and time in a large sample of 6–8 year-old children in Uruguay by examining how individual differences in the precision of number and time estimation correlate with school mathematics performance. Over four testing days, each child completed an approximate number discrimination task, an approximate time discrimination task, a digit span task, and a large battery of symbolic math tests. We replicate previous reports showing that symbolic math abilities correlate with approximate number precision and extend those findings by showing that math abilities also correlate with approximate time precision. But, contrary to approximate number and time sharing common representations, we find that each of these dimensions uniquely correlates with formal math: approximate number correlates more strongly with formal math compared to time and continues to correlate with math even when precision in time and individual differences in working memory are controlled for. These results suggest that there are important differences in the mental representations of approximate number and approximate time and further clarify the relationship between quantity representations and mathematics.

1. Introduction

What is the source of our intuitions about number? Recent work in cognitive development has focused on young children's ability to quickly and intuitively represent the number of items in a collection through the Approximate Number System (ANS; Dehaene, 2009; Halberda & Feigenson, 2008; Halberda, Mazzocco, & Feigenson, 2008; Halberda & Odic, 2014; Feigenson, Dehaene, & Spelke, 2004; Odic, Hock, & Halberda, 2014; Odic, Libertus, Feigenson, & Halberda, 2013). The multimodal ANS provides us with a rough and noisy sense of number, such as when guessing how many people are sitting in a lecture hall or how many items are in our shopping basket.

The ANS is characterized by three empirical signatures (Halberda & Odic, 2014; ; Feigenson et al., 2004). First, discrimination performance in the ANS is ratio-dependent (i.e., obeys Weber's law): discriminating a collection of 10 items from 5 items (a ratio of 2.0) is much easier than discriminating a collection of 10 items from 9 items (a ratio of 1.11). The precision with which an individual can successfully discriminate difficult ratios is often quantified through the Weber fraction (w), and theoretically corresponds to the amount of noise in the underlying ANS representations (Cordes, Gallistel, Gelman, & Latham, 2007; Halberda & Odic, 2014; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004). Second, there are large individual differences in ANS precision, and children's ANS continues to improve from birth onward, peaking around age 30 (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Odic, Libertus, Feigenson, & Halberda, 2013; Piazza et al., 2010). Finally, the ANS has been localized in both the human brain and in non-human animals to a region of the intraparietal sulcus (IPS; Dehaene, Piazza, Pinel, & Cohen, 2003; Nieder, 2005, 2012; Piazza et al., 2010; Roitman, Brannon, & Platt, 2007); physiological modulations of the IPS can, for example, enhance ANS discrimination (Cappelletti et al., 2013).

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Researchers have also focused on the relationship between the ANS and formal mathematical abilities.[1] Individual differences in ANS precision show a small but significant relationship with formal math, including in preschoolers (Feigenson, Libertus, & Halberda, 2013; Libertus, Feigenson, & Halberda, 2011; Starr, Libertus, & Brannon, 2013) and adults (DeWind & Brannon, 2012; Libertus, Odic, & Halberda, 2012; Lyons & Beilock, 2011). Temporary modulations of the ANS can also selectively enhance or impair subsequent math performance (Hyde, Khanum, & Spelke, 2014; Park & Brannon, 2013; Wang, Odic, Halberda, & Feigenson, under review). Finally, individuals with math learning disabilities also show impaired ANS precision (Mazzocco, Feigenson, & Halberda, 2011; Piazza et al., 2010). This work, though not unchallenged (e.g., De Smedt, Noël, Gilmore, & Ansari, 2013), suggests that our basic intuitions about math may emerge, in part, from a universal and ontologically ancient core cognitive system, and that intervention methods that improve the ANS may also help children in acquiring formal math concepts.

But the ANS is not alone in showing these behavioral and neural signatures. Many other dimensions, including surface area, time, density, weight, brightness, and line length, also obey Weber's law (Cantlon, Platt, & Brannon, 2009; Cheng, Srinivasan, & Zhang, 1999; Feigenson, 2007; Gescheider, 1997; Meck & Church, 1983; Möhring, Libertus, & Bertin, 2012; Stone & Bosley, 1965), develop with age (Brannon, Lutz, & Cordes, 2006; Droit-Volet, Clément, & Fayol, 2008; Odic, Le Corre, & Halberda, 2015; Odic et al., 2013), and are localized in the IPS (Cantlon et al., 2009; Castelli, Glaser, & Butterworth, 2006; Pinel, Piazza, Le Bihan, & Dehaene, 2004; Tudusciuc & Nieder, 2007). For example, transcranial noise stimulation of the IPS modulates both number and time discrimination (Cappelletti et al., 2013), and 6-month-old infant's Weber fractions for surface area discrimination appear identical to Weber fractions for number and time discrimination (Brannon et al., 2006; Feigenson, 2007). These similarities between distinct dimensions have led many researchers to suggest that number, time, and space are all represented by common mechanisms — a domain-general "generalized magnitude system" (Bueti & Walsh, 2009; Cantlon et al., 2009; Lourenco & Longo, 2010; Vicario, 2013; Walsh, 2003). Additional evidence for the generalized magnitude system comes from correlations of Weber fractions across dimensions (e.g., time and number; Meck & Church, 1983, but see Droit-Volet et al., 2008), and from persistent congruency and interference effects between quantities, whereby manipulation of one dimension affects discrimination performance of another (Barth, 2008; Dakin, Tibber, Greenwood, Kingdom, & Morgan, 2011; Gebuis & Reynvoet, 2012; Hurewitz, Papafragou, Gleitman, & Gelman, 2006; Leibovich & Henik, 2013; Lourenco & Longo, 2010; Szucs, Nobes, Devine, Gabriel, & Gebuis, 2013; Wood, Willmes, Nuerk, & Fischer, 2008).

Although researchers have frequently invoked the generalized magnitude system as an explanation for the behavioral and neural commonalities among quantity representations, it remains unclear what the shared mechanism between number and other dimensions might be. There are at least three (non-mutually exclusive) possibilities. First, quantity representations could share low-level sensory encoding processes. Dakin et al. (2011), for example, suggest that the number and density are both encoded through low spatial-frequency filters; hence, modulations of density (and thus of low spatial-frequency) will simultaneously impact number discrimination. Second, various dimensions might all be represented on an identical domain-general quantity scale and by identical sets of neurons that code for "more" or "less" of any and every dimension (Bueti & Walsh, 2009; Lourenco & Longo, 2010; Tudusciuc & Nieder, 2007); in this case, representations for, e.g., time and number, will show identical Weber fractions, identical individual and developmental differences, and will be equivalently impacted by any modulation of the IPS. Finally, different dimensions may share common decision making or comparison computations, such as determining a threshold before the response is initiated; as a result, quantity representations may compete for behavioral responses and interfere with one another (DeWind & Brannon, 2012; Hurewitz et al., 2006; Van Opstal, Gevers, De Moor, & Verguts, 2008) and bottlenecks on attentional, memory, or decision making processes may result in similar Weber fractions across dimensions. Droit-Volet et al. (2008), for example, find that time and number Weber fractions only correlate when both dimensions are presented sequentially, suggesting that attentional and memory processes may be responsible for their correlation.

The existing evidence has not determined the best explanation for the common behavioral and neural signatures between number, time, and space (though most researchers seem to prefer the shared representations account). Recently, an increasing number of studies have attempted to dissociate quantity representations by examining how they relate to other cognitive abilities, such as affect (Droit-Volet, 2013; Young & Cordes, 2013) or formal mathematics (DeWind & Brannon, 2012; Lourenco, Bonny, Fernandez, & Rao, 2012). If, for example, the ANS correlates with formal math independently from non-numeric dimensions such as surface area, then we would have evidence for an important degree of independence between these dimensions. DeWind and Brannon (2012) recently found that while number and line-length discrimination correlate in precision in adults, only number correlates with formal math (as assessed by SAT scores). Similarly, Lourenco et al. (2012) found that while number and cumulative surface area correlate in precision amongst adults, individual differences in the ANS uniquely correlate with arithmetic math problems, while individual differences in cumulative area precision uniquely correlate with geometric math problems. Combined, this work suggests important distinctions in the representations of number and spatial extent and their relationship to formal math, and further implies that the commonality between these dimensions is unlikely to be due to both number and spatial extent being represented on an identical scale.

A similar kind of approach has been used to differentiate the ANS from approximate time perception. Time perception provides a useful case-study because its relationship to the ANS is still very actively debated. Meck and Church (1983) famously proposed that both time and number are encoded by an accumulating pacemaker mechanism, and found that amphetamine administration equally affects time and number perception in rats. Furthermore, affective stimuli, such as sad or happy faces, appear to impact both time and number equally (Droit-Volet, 2013), there are known shared neural substrates for time and number perception (Dormal, Dormal, Joassin, & Pesenti, 2012), and these dimensions show mapping and interference effects (Bueti & Walsh, 2009; Müller & Schwarz, 2008; Oliveri et al., 2008). Focusing on children and adults with math learning disabilities, previous work has shown mixed results in dissociating time from number perception. For example, Cappelletti, Freeman, and Butterworth (2011) find that time perception is not affected in adults diagnosed with dyscalculia. On the other hand, both Hurks and Loosbroek (2012) and Vicario, Rappo, Pepi, Pavan, and Martino (2012) find that children with math learning disabilities show abnormal time estimation and production. Combined, the existing work does not conclusively show evidence for or against time and number being part of a single generalized magnitude system.

The existing work on time, number, and their relationship to formal mathematics leaves open the possibility that quantity representations diverge and differentiate with development, especially as children acquire formal math concepts from preschool onward. Additionally, previous work has only tested children with math learning disabilities and used small sample sizes. Here, we examine the relationship between the ANS, time perception, and a series of formal math tests in a large sample of children tested at schools in Uruguay. By examining the

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[1] In this paper, we refer to "formal" math in the sense of symbolic, abstract, school-taught mathematics, rather than differentiating between more informal math skills, such as addition and counting, and more formal math skills, such as word problems (e.g., see Libertus et al., 2013).