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LIDFT method with classic data windows and zero padding in multifrequency signal analysis

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ABSTRACT

This paper presents an advanced discrete Fourier transform linear interpolation method (LIDFT) using classic data windows and a zero padding technique that is well understood from its common use in spectrum analysis with FFT. This advancement of the LIDFT method allows for substantial improvement of its accuracy without deteriorating the condition of the initial location for sinusoidal components in the spectrum, as required in the traditional LIDFT method. Application of classic data windows instead of parametric data window specifically dedicated to the LIDFT method provides numerous opportunities for using this method in multifrequency signal analysis.

1. Introduction

The method of linear interpolation of discrete Fourier transform (LIDFT) approximates the window frequency characteristic with appropriate linear functions in order to obtain the linear matrix equation, from which amplitudes, frequencies and phases of component oscillations can be derived [1–3]. Introducing classic data windows and a zero padding technique to the LIDFT method shows, even more clearly, that the LIDFT method combines beneficial properties from two groups of analytical methods for amplitudes, frequencies and phases of component oscillations in a multifrequency waveform. One group applies a data window [4], the discrete Fourier transform, and the interpolation of the local maxima of the obtained spectrum (a suitable comparison of these methods is given in [5–7]). The other group covers methods based on the Prony method and other correlation methods founded on properties of the signal autocorrelation matrix [10–12].

In this paper, the main goal of combining the LIDFT method [1–3], a zero padding technique and classic data windows into one extended LIDFT method is to improve method accuracy in comparison to other existing spectrum interpolation methods [1–9]. What follows are the most important facts to consider when justifying such an approach, including references to results from previous research work.

1. Sampling a multifrequency signal (i.e., a sum of sinusoidal components), applying a data window and calculating the DFT spectrum (most often with FFT algorithm) leads to a discrete spectrum wherein, when the simplest analysis is applied, the assumption is made that the local maxima of that spectrum are used to determine the parameters of the sinusoidal components (amplitude, frequency and phase). The discrete nature of the resultant spectrum causes a maximum estimation error for the component amplitude of [4]: −3.92 dB (i.e., −36%) when a rectangular window is used, and an error between [−2.2 dB, −0.8 dB] (i.e., [−22%, −9%]) for the remaining windows [4]. To reduce this error, various

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DFT spectrum interpolation methods are used. An additional error is that caused by the spectrum leakage effect; however, this constituent is neglected in all interpolation methods known to date, except the LIDFT method and multifrequency weighted interpolated DFT (WIDFT) methods [7–9]. When the dynamics of the best A/D converters (over 16 bits) are fully utilised, spectrum leakage cannot be neglected for precision measurements of complex oscillations (e.g., in mechanics, acoustics, speech and audio, etc.). The uniqueness consists in considering the spectrum leakage in the equations used in the LIDFT method, allowing the LIDFT method to be used where former interpolation methods have failed. The method is general in nature, with the only additional assumption being that the signal is multifrequency.

2. In order to avoid spectrum leakage and spectrum interpolation, methods based on the Prony method and correlation methods [10–12] are often applicable. However, problems with complicated and non-linear procedures of estimation for component frequencies must then be addressed. As an example, in the least squares Prony method, component frequencies are determined from the locations of zeros in a high-order polynomial. The advantage of the LIDFT method presented here is in combining the advantages of the DFT method for spectrum calculation (fast FFT algorithm and its simplicity) and the advantages of techniques based on the Prony method and correlation methods (accuracy and application of linear matrix equation) while considerably minimising their adverse features (spectrum leakage for DFT and nonlinearity of relations for determining component frequencies).
3. Using data windows in calculating DFT is a well-known spectrum analysis technique that minimises the spectrum leakage effect introduced by a finite measurement time. The most important parameters of data windows include the main lobe width, the suppression of the side lobes (these parameters are very well illustrated by the frequency characteristic of the data window) and the noise properties of the window defined by the *equivalent noise bandwidth* (ENBW) coefficient [4]. Such windows, used in standard DFT analysis, are, for the purpose of this paper, considered classic windows. The special window refers to this introduced to the LIDFT method in Refs. [1–3]. The novelty of this paper is using classic data windows in LIDFT instead of a special dedicated window [1–3], thus reaching good results as long as the zero padding technique is also used.
4. The zero padding technique consists in R-times padding the N signal samples (multiplied by the data window) by zeros, prior to the DFT calculation. This is simply a numerical operation (as zero samples give no new information about the signal), but it allows more accurate sampling of the signal spectrum because, instead of N spectrum samples, $R \cdot N$ samples of the same spectrum are available. The other spectrum parameters, such as the spectrum leakage level (of the main lobe width or suppression of the side lobes), and signal-to-noise ratio in the spectrum, remain unchanged. The zero padding technique is a simple method of spectrum interpolation as it allows for more accurate positioning of the local maxima for determining spectrum components. Application of the zero padding technique in the LIDFT method is the essence of this paper. This combination reduces the values of errors and allows the use of classic data windows.
5. All spectrum interpolation methods can be considered to be a post-processing step of the derived DFT spectrum wherein a specific interpolation method is applied for selected spectrum samples (where we let the indices of selected samples create set S). For the zero padding method, the set S includes one element for each spectrum component, while two or more elements per each spectrum component are used in the remaining interpolation methods. Selection of the set S corresponds to a preliminary location of components in the spectrum and is most often made on the basis of analysing the local maxima of the DFT spectrum (with no absolute certainty that the component locations are correct) or additional *a priori* information about the signal. The LIDFT method likewise requires one to provide the initial location of components in a spectrum, but it also allows for these locations to be subsequently corrected in an iteration procedure. The novelty of the technique presented in this paper lies in the determination of the condition of proper convergence of such an iteration – the data window used must have a non-negative frequency characteristic (such as a triangular window).
6. Metrological analysis of the first version of the method [3] has been run for the window dedicated to this method, and no correlation has been made between the results obtained and frequency characteristic of this data window. This window has a poor frequency characteristic and fails to meet the condition of having a non-negative frequency characteristic. Hence, in practice, it is disqualified from the extended version of the LIDFT method as presented in this paper. Thus, there is no need to compare the results obtained in this paper with those in [3]. The advantage of this paper is the innovative methodology of analysing the LIDFT method accuracy. We show a close relation between LIDFT method accuracy and the frequency characteristic of the data window in use (the higher the suppression of side lobes, the better the accuracy of the LIDFT method) and the number of padded zeros. This allows for the use of the standard window analysis described in [4], including an assessment of the noise properties of the window by the ENBW coefficient.
7. The data windows used in DFT calculations allow one to estimate the spectrum components the amplitude of which is clearly higher than spectrum leakage due to adjacent components. The originality of the LIDFT method is that it allows one to estimate spectrum components when the amplitude may be lower than the spectrum leakage from adjacent components – but it is essential that this amplitude is considerably larger than the error of the spectrum approximation by linear functions. This paper is innovative in allowing the application, in the LIDFT method, of classic data windows for which the spectrum leakage level is already relatively low and the approximation error obtained by linear functions is even lower. This means that the