The negations of conjunctions, conditionals, and disjunctions

Sangeet Khemlani [a,*], Isabel Orenes [b], P.N. Johnson-Laird [c,d]

[a] *Naval Research Laboratory, United States*
[b] *Universidad de la Laguna, Tenerife, Spain*
[c] *Princeton University, United States*
[d] *New York University, United States*

ABSTRACT

How do reasoners understand and formulate denials of compound assertions, such as conjunctions and disjunctions? A theory based on mental models postulates that individuals enumerate models of the various possibilities consistent with the assertions. It therefore predicts a novel interaction: in affirmations, conjunctions, *A and B*, which refer to one possibility, should be easier to understand than disjunctions, *A or B*, which refer to more than one possibility; in denials, conjunctions, *not(A and B)*, which refer to more than one possibility, should be harder to understand than disjunctions, *not(A or B)*, which do not. Conditionals are ambiguous and they should be of intermediate difficulty. Experiment 1 corroborated this trend with a task in which the participants selected which possibilities were consistent with assertions, such as: *Bob denied that he wore a yellow shirt and he wore blue pants on Tuesday*. Experiment 2 likewise showed that participants' own formulations of verbal denials yielded the same trend in which denials of conjunctions were harder than denials of conditionals, which in turn were harder than denials of disjunctions.

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1. Introduction

To deny an affirmative assertion is to negate it, and negation serves an important function in natural language (e.g., Horn, 2001) and in logic (e.g., Aristotle, 1984; Quine, 1974). Negation is also important in psychology since it is an abstract concept with a meaning outside any sensory modality (cf. Barsalou, 1999; Glenberg, Robertson, Jansen, & Glenberg, 1999; Hald, Hocking, Vernon, Marshall, & Garnham, 2013). Early psychological studies of negation focused on the interpretation of negative sentences in part because the then theory of transformational grammar introduced negation by way of a transformation (Klima, 1964). Their principal discovery, however, was semantic. Not only were negative assertions, such as, "The circle is not above the triangle", harder to verify than their affirmative counterparts, but there was an interaction between the polarity of an assertion (affirmative or negative) and its truth value (true or false): true affirmatives were easier to verify than false affirmatives, whereas true negatives were harder to verify than false negatives (Wason & Jones, 1963). This discovery led to the formulation of various information-processing theories of negation (e.g., Clark & Chase, 1972; Dale & Duran, 2011; Kaup, Zwaan, & Lüdtke, 2007; Orenes, Beltrán, &

Santamaría, 2014; Wason & Johnson-Laird, 1972). What studies have not considered, however, is the negation of different sorts of compound assertion, such as conjunctions ("and"), conditionals ("if_ then_"), and disjunctions ("or"). The present paper presents an investigation of them.

We carried out various preliminary studies, both online and face-to-face, which showed that naïve individuals – those who have not studied logic – have difficulty in understanding the task of "negating" assertions. For example, when we asked participants to list what was impossible given the "negation" of compound assertions, their performance was almost at chance. We therefore framed our experiments using a concept that they did understand: the denial of assertions. In linguistics, negation is a syntactic concept with semantic consequences. As Aristotle argued (see *De Interpretatione* in Aristotle, 1984, Vol. 1), negations contradict the negated assertion, i.e., they reverse its truth value: the negation of a true assertion is false, and the negation of a false assertion is true. But, negation can apply to constituents of sentences, and to interrogatives and imperatives. In contrast, denial is a speech act in which speakers correct assertions, not questions or requests, by negating affirmatives or unnegating negatives. In the context of our experiments, no difference exists between the following two sorts of instruction: please formulate a negation of this sentence, and please formulate a denial of this sentence, except that naïve individuals are much less likely to be confused by the latter instruction, because "negation" sounds like a syntactic command rather than a semantic one. Hence, in what follows, we will treat "denial" and "negation" as interchangeable.

* Corresponding author at: Navy Center for Applied Research in Artificial Intelligence, Naval Research Laboratory, 4555 Overlook Ave. SW, Washington, DC 20375, United States. Tel.: +1 571 339 931.
E-mail address: skhemlani@gmail.com (S. Khemlani).

The paper begins with an account of negation from a logical standpoint, which we have based on Rips's (1994) psychological theory. Next, the paper describes a contrasting theory based on mental models. It then reports two experiments designed as crucial tests of the theories' predictions. Finally, it relates the results of the experiments to a general account of negation (Khemlani, Orenes, & Johnson-Laird, 2012).

1.1. The negation of compounds in logic

How do individuals understand the consequences of the negation, or denial, of compound assertions? If they know De Morgan's laws for interrelating the negations of conjunctions and disjunctions, they can apply the laws to infer a conclusion expressing the correct negation. These laws are embodied in Rips's (1994, p. 112 et seq.) PSYCOP theory, as follows:

1. NOT (P AND Q) implies (NOT P) OR (NOT Q)
2. NOT (P OR Q) implies (NOT P)
3. NOT (P OR Q) implies (NOT Q)

In these rules, OR is an inclusive disjunction, which allows that both disjuncts can be true.

The rules can be used to work forwards from a premise to draw a conclusion. Rule (1) can also be used to work backwards from a given conclusion, but PSYCOP includes a single rule that combines (2) and (3) in order to work backwards to prove that a given conclusion, (NOT P) AND (NOT Q), follows from the premise (NOT P OR Q). To illustrate how De Morgan's rules work, suppose that you are asked for the consequences of the assertion:

4. It's not the case that Pat entered the room and she saw Viv.

Your first step is to grasp that its logical form is NOT (P AND Q), where P signifies *Pat entered the room* and Q signifies *Pat saw Viv*. Your second step is to find and to apply the corresponding formal rule of inference (1) to yield the conclusion: (NOT P) OR (NOT Q). And your final step is to restore the content as the values of the variables in the conclusion:

5. Pat didn't enter the room or she didn't see Viv.

PSYCOP predicts that it should be more difficult to determine the consequences of the negation of a disjunction, that is, to work forwards to a conclusion from:

6. It's not the case that Pat entered the room or she saw Viv.

You must use both rules (2) and (3), and the rule for forming a conjunction of their respective consequences. It follows that the denial of a conjunction should be easier to grasp than the denial of a disjunction.

The following implication is valid in logic:

7. NOT (IF P THEN Q) implies P AND NOT-Q

Some proponents of formal rules of inference appear to accept such a rule. For example, Beth and Piaget (1966, p. 181) wrote that given a hypothesis of the form, *if p then q*, individuals should try to refute it by searching for a counterexample, *p and not-q*. But, rule (7) strikes many people, including Rips, as not intuitive, and so he excludes it from PSYCOP. It follows, as Rips proves, that PSYCOP cannot make the following sort of inference:

8. It's not the case that if Pat entered the room then she saw Viv.
So, Pat entered the room and she didn't see Viv.

Such inferences could be proved only if such rules as (7) are added to the system (Rips, 1994, p. 128). Presented with the inference in (8), PSYCOP itself halts but without a proof that the conclusion follows from the premise. In summary, formal rules of inference lead to the psychological prediction that the denial of a disjunction should be harder than the denial of a conjunction, and the denial of a conditional should be hardest of all, if not impossible.

1.2. Mental models and the negation of compounds

The theory of mental models – the "model" theory for short – differs in several ways from an account based on formal rules of inference. The model theory neither extracts logical forms nor applies formal rules of inference to them. Instead, the model theory postulates that individuals

grasp the significance of an assertion when they know the *possibilities* to which it refers (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991). The mind constructs mental models of these possibilities. We now explore how the theory treats various compound assertions and their denials.

A conjunction, such as (9a), refers to a single possibility in which both clauses hold, whereas its denial (9b) refers to three possibilities:

9a. Pat entered the room and she saw Viv.
9b. It's not the case that both Pat entered the room and she saw Viv.

¬P	¬V
¬P	V
P	¬V

We list the three possibilities for (9b) on separate rows and abbreviate them as follows:

where '¬' denotes negation, *P* stands for *Pat entered the room*, and *V* stands for *Pat saw Viv*. (We use letters in these diagrams for convenience; in reality, people build models of the world.) In contrast, consider a disjunction and its negation:

10a. Pat entered the room or she saw Viv.

10b. It's not the case that Pat entered the room or she saw Viv.

P	¬V
¬P	V
P	V

¬P	¬V

Given an inclusive interpretation, the disjunction (10a) refers to three possibilities:

Its negation (10b) refers to only one possibility:

Given an exclusive interpretation, however, both the affirmation of the disjunction and its denial refer to two possibilities: *P and V* shifts from an affirmative possibility to a negative one. As the preceding examples illustrate, the negation of the models of an affirmative assertion yield the models of the corresponding negative assertion, where the negation of the models are their complement in the set of all possible models based on the relevant atomic propositions.

Conditional assertions are more complicated and more controversial than the preceding compounds (see, e.g., Evans, 2007; Handley, Evans, & Thompson, 2006; Johnson-Laird, Byrne, & Girotto, 2009). Unlike conjunctions and disjunctions, conditional assertions, such as (11), contain a subordinate clause (the if-clause) and a main clause (the then-clause),

11. If she entered the room then Pat saw Viv.

One sign of a subordinate clause is that, as in this example, a pronoun can refer forwards to the same referent as a noun phrase in the subse-

P	V
…	

quent main clause. Such a "cataphorical" reference, however, is not possible from one main clause to another, and so "she" doesn't refer to the same individual as "Pat" in this example: