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Learning to focus on number

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ABSTRACT

With age and education, children become increasingly accurate in processing numerosity. This developmental trend is often interpreted as a progressive refinement of the mental representation of number. Here we provide empirical and theoretical support for an alternative possibility, the filtering hypothesis, which proposes that development primarily affects the ability to focus on the relevant dimension of number and to avoid interference from irrelevant but often co-varying quantitative dimensions. Data from the same numerical comparison task in adults and children of various levels of numeracy, including Mundurucú Indians and western dyscalculics, show that, as predicted by the filtering hypothesis, age and education primarily increase the ability to focus on number and filter out potentially interfering information on the non-numerical dimensions. These findings can be captured by a minimal computational model where learning consists in the training of a multivariate classifier whose discrimination boundaries get progressively aligned to the task-relevant dimension of number. This view of development has important consequences for education.

1. Introduction

During development, children become increasingly precise in making numerical judgments (Halberda & Feigenson, 2008). The evidence for this change comes primarily from numerosity comparison or discrimination tasks, where participants are asked to point, without counting, to the numerically larger (or smaller) of two sets, or to decide whether two sets contain the same number of items. Performance on such tasks depends on the logarithm of the ratio (log ratio) of the two numerosities, according to Weber's law (Dehaene, 2007). Studies in naïve non-human animals (Agrillo, Dadda, Serena, & Bisazza, 2008; Jordan, Brannon, Logothetis, & Ghazanfar, 2005; Rugani, Regolin, & Vallortigara, 2011; Viswanathan & Nieder, 2015) and human newborns (Izard, Sann, Spelke, & Streri, 2009) indicate that number, like many other quantitative dimensions of the environment, is immediately available, even in the absence of training. However, the precision of numerical discrimination is initially low (newborns discriminate sets only when they differ by 300%), and it improves progressively during development (adults eventually differentiate small 15–20% numerical changes) (Halberda & Feigenson, 2008). Recent investigations indicate that while brain maturation is responsible for this evolution during the first years of life, formal education plays a key role in increasing numerical discrimination performance later on (Guillaume, Nys, Mussolin, & Content, 2013; Piazza, Pica, Izard, Spelke, & Dehaene, 2013; Nys et al., 2013).

The most straightforward explanation for this behavioral improvement, hereafter called the *sharpening* hypothesis, assumes that maturation and formal education progressively sharpen the internal representation of numerosity, see Fig. 1B. The intraparietal cortex of both humans and macaques has been identified as a key node for the neural representation of numerosity (Piazza & Eger, 2015), and this hypothesis holds that the tuning curves of neurons in this region get progressively sharper. This idea recently received partial support by two fMRI studies investigating numerosity coding precision in the intraparietal sulcus (hereafter IPS) of adults and young preschoolers tested with an identical adaptation paradigm: the pattern of fMRI responses to numerically deviant stimuli, a proxy for "numerosity tuning functions", were sharper in adults (Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004) compared to preschoolers (Kersey & Cantlon, 2017), mirroring their higher accuracy in numerical discrimination precision. However, because the BOLD signal has limited temporal resolution, it remains possible that the brain activation in this paradigm reflected the effect of a post-perceptual attentional amplification rather than the initial encoding of numerosity.

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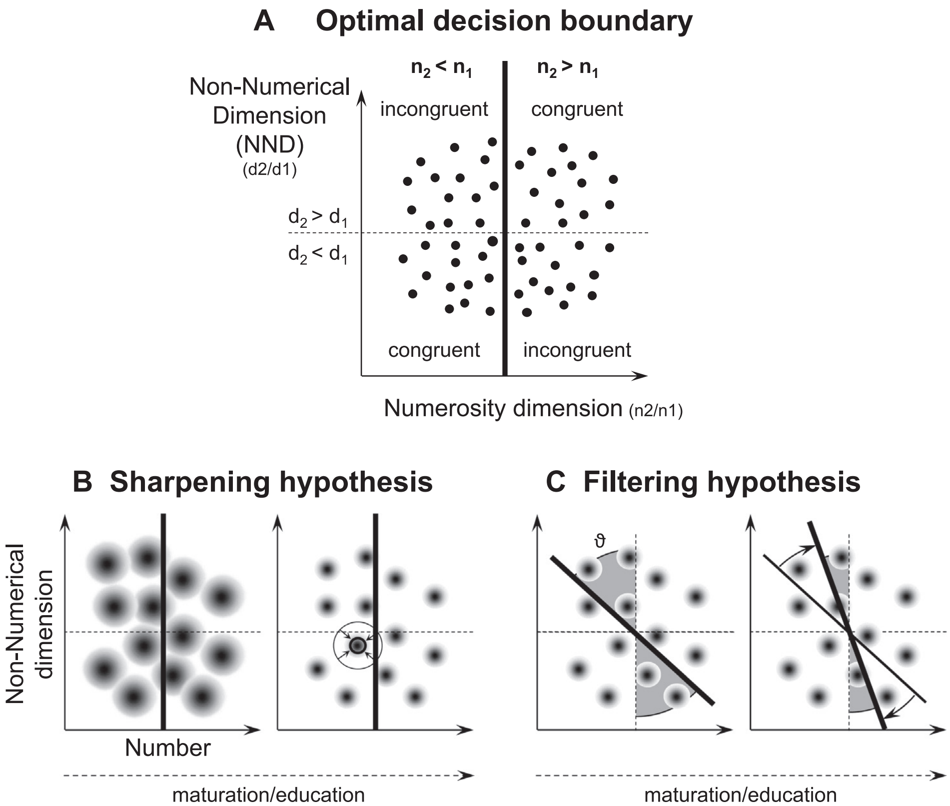

Fig. 1. Two potential sources of errors and two theoretical accounts of the increasing precision of numerical abilities with age and education. The task studied in the present paper requires comparing pair of sets of different numbers (n1 and n2) each characterized by different non-numerical dimensions (such as total occupied area or individual items size, d1 and d2), and to choose the set with the largest number ignoring the non-numerical dimensions. In panel A, each dot represents one individual trial that is a specific combination of two numbers (their log ratio varying along the horizontal axis) and their relative non-numerical features (their log ratio varying along the vertical axis). The vertical line indicates the optimal decision boundary for such a number comparison task. Dots close to the decision boundary represent trials where the two numerosities vary little, and their distance increases as we move away from it. Panel B and C represent the two potential sources of errors and of developmental changes, referred to as the "sharpening" and the "filtering" hypotheses respectively. In these panels the width of the dots represents the noise of internal representations of number and of another non-numerical dimension (NND). According to the sharpening model (B), representations are initially highly noisy and they become more precise (sharper) with age and education. Sharpening predicts an overall reduction in error rates, particularly for stimuli close to the decision boundary, but not necessarily a reduction in the congruity effect: error rate should equally decrease in the congruent and incongruent pairs. According to the filtering model (C), numerical development involves an increasing capacity to focus on the relevant dimension and to filter out irrelevant non-numerical dimensions, with no concurrent change in the precision of the underlying representations. Such a development is illustrated here as a progressive rotation of the decision boundary towards the optimal vertical line, thus a reduction of the angle (ϑ) between the actual decision slope and the optimal one. Filtering predicts a reduction of the congruity effect in that error rates should solely decrease in the incongruent conditions (the shaded area shrinks).

Conceptually, however, developmental improvements in numerical judgement may also result from an improved ability to selectively attend to the representation of numerosity and amplify the contribution of numerosity to perceptual judgement while ignoring other quantitative information (average item size, density, total occupied area) that is also automatically extracted from sets of multiple items. According to this *filtering* hypothesis (see Fig. 1C), children get progressively better at teasing apart numerical from non-numerical quantitative variables when confronted with sets. Evidence suggests that already at an early age, children spontaneously estimate the variables of numerosity, size, and surface area (Cordes & Brannon, 2008, 2011). During development, the decision system would learn to focus on numerosity and to avoid interference from other continuous magnitudes, thus resulting in an increasingly accurate judgment. The existence of a congruity effects in numerical processing fits squarely with the filtering hypothesis. When asked to choose the numerically larger of two sets, human adults are less accurate when the size of the items, or the inter-item distance is incongruent with number, than when it is congruent (Gebuis & Reynvoet, 2012). Congruity effects are thought to arise from the fact that numerical and non-numerical dimensions are encoded in overlapping sectors of parietal cortex, and in some cases, by the very same neurons (Harvey, Fracasso, Petridou, & Dumoulin, 2015; Pinel, Dehaene, Riviere, & LeBihan, 2001; Tudusciuc & Nieder, 2009). Because of this overlap, brain areas downstream of those representing numerical and non-numerical dimensions may be confronted with the same problem that confronts multivariate classifiers, namely the identification of relevant dimensions in a highly multidimensional set of neuronal responses (King & Dehaene, 2014, box2).

Sharpening and filtering are not necessarily mutually exclusive learning mechanisms: both may jointly occur during development/education. However, they are qualitatively different. The former affects the precision of the representation (see Fig. 1B), while the latter affects the effectiveness of the decision system at discarding task-irrelevant representations (see Fig. 1C). Indeed, the two hypotheses make rather different predictions of the developmental time course of performance. As illustrated in Fig. 1B, if sharpening is the only mechanism, then there should be an overall reduction in error rates, particularly for stimuli close to the decision boundary, but not necessarily a reduction in the congruity effect: decreasing the noise without changing the decision boundary should result in increases in accuracy in both trials where number is incongruent with non-numerical dimensions and trials where number and the non-numerical dimensions are congruent. If only filtering is at work (see Fig. 1C), on the other hand, learning should differentially affect the congruent and incongruent trials: progress should be mostly observed on incongruent trials, but it should be absent on congruent trials. If there is only filtering, it is even possible that, in the course of learning, children would perform increasingly *worse* on congruent trials, as they would lose the benefit of a reliance on correlated helping variables. Such a behavior would clearly speak against the sharpening model, which would be unable to accommodate a decrease in performance. A third possibility is that, because sharpening and filtering are not mutually exclusive, they both occur during development: this would result in improvements occurring in both congruent and incongruent conditions, but more so in the incongruent conditions.

To test those predictions, we re-analyzed a large set of previously published psychophysical data where subjects of different ages and levels of numeracy were engaged in a common numerosity comparison task. Contrary to most previous research (Bugden & Ansari, 2015; Gilmore et al., 2013; Szucs, Nobes, Devine, Gabriel, & Gebuis, 2013), here we varied the degree of congruity between numerical and non-