Cognition 180 (2018) 59–81

Contents lists available at ScienceDirect

Cognition

journal homepage: www.elsevier.com/locate/cognit

Meaning before order: Cardinal principle knowledge predicts improvement in understanding the successor principle and exact ordering

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ARTICLE INFO

Keywords:
Number
Cardinal principle
Successor function
Successor principle
Exact ordering

ABSTRACT

Learning the *cardinal principle* (the last word reached when counting a set represents the size of the whole set) is a major milestone in early mathematics. But researchers disagree about the relationship between cardinal principle knowledge and other concepts, including *how counting implements the successor function* (for each number word N representing a cardinal value, the next word in the count list represents the cardinal value N + 1) and *exact ordering* (cardinal values can be ordered such that each is one more than the value before it and one less than the value after it). No studies have investigated acquisition of the successor principle and exact ordering over time, and in relation to cardinal principle knowledge. An open question thus remains: Is the cardinal principle a "gatekeeper" concept children must acquire before learning about succession and exact ordering, or can these concepts develop separately? Preschoolers ($N = 127$) who knew the cardinal principle (CP-knowers) or who knew the cardinal meanings of number words up to "three" or "four" (3–4-knowers) completed succession and exact ordering tasks at pretest and posttest. In between, children completed one of two trainings: counting only versus counting, cardinal labeling, and comparison. CP-knowers started out better than 3–4-knowers on succession and exact ordering. Controlling for this disparity, we found that CP-knowers improved over time on succession and exact ordering; 3–4-knowers did not. Improvement did not differ between the two training conditions. We conclude that children can learn the cardinal principle without understanding succession or exact ordering and hypothesize that children must understand the cardinal principle before learning these concepts.

1. Introduction

Preschool numeracy lays a critical foundation for later mathematics. Indeed, kindergarten-entry math predicts math achievement through high school (Watts, Duncan, Siegler, & Davis-Kean, 2014), and is a better predictor of later achievement in math and reading than kindergarten-entry reading, attention, or socio-emotional skills (Duncan et al., 2007). Understanding the development of early mathematical skills is therefore critical for supporting long-term achievement in this domain. One noteworthy step in a preschool child's development is acquisition of the *cardinal principle*: that the last word reached when counting a set represents the size of the whole set (Gelman & Gallistel, 1978). Although this accomplishment may seem quite easy to adults, knowing the cardinal principle is a major milestone for a preschooler, leading to many new numerical competencies (e.g., Le Corre, 2014; Le Corre, Van de Walle, Brannon, & Carey, 2006; Mix, 2008; Sarnecka & Wright, 2013; Wynn, 1992).

1.1. Developmental trajectory of cardinal number knowledge

A large body of research has established that children can count a set of objects correctly by age 3 (Fuson, 1988; Gelman & Gallistel, 1978; Wynn, 1990). However, children do not initially understand the meaning of the count list, nor the quantities represented by each number word. Children learn the cardinal meanings of the number words (e.g., that "two" means a set of two objects) one at a time, and in order (e.g., Le Corre & Carey, 2007; Sarnecka & Lee, 2009; Wynn, 1992). First, children can comprehend and produce sets of "one" object on request, but fail to accurately produce larger sets of objects ("one-knowers"). A few months later, children can comprehend and produce sets of "two", but not higher numbers ("two-knowers"). Children go through the same stages for "three" and "four" ("three-knowers" and

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https://doi.org/10.1016/j.cognition.2018.06.012
Received 1 July 2016; Received in revised form 7 June 2018; Accepted 18 June 2018

"four-knowers"), and then learn the cardinal principle (Wynn, 1992). This developmental sequence is protracted, taking between 1 and 2 years from when a child becomes a one-knower to when the child learns the cardinal principle (Sarnecka, Goldman, & Slusser, 2015; Wynn, 1992). Learning the cardinal principle is a major achievement at this age. After learning the cardinal principle, children can typically produce sets of any size that they can count to (Wynn, 1992); correct errors made when producing a set by spontaneously counting (Le Corre et al., 2006); match dissimilar sets based on set size (Mix, 2008); and recognize equi-numerosity (that only sets that can be placed in one-to-one correspondence have the same number) (Sarnecka & Wright, 2013).

But how do children conceptualize the relationships *between* numbers as they learn the values of individual numbers and number words over this protracted period? That is, as children learn the cardinal meanings of the numbers "one," "two," and "three," do they learn them as largely separate pieces of knowledge, or do they also consider what makes "two" different from "one" and "three"? This concept, which we will call "exact numerical relations," has at least two related, but separable parts: (1) knowledge of *how counting implements the successor function*—for each number word N representing a cardinal value, the next word in the count list represents the cardinal value N + 1 (for brevity we will call this the "the successor principle" or simply "succession"); (2) knowledge of *exact numerical order*—that cardinal values themselves (not just the words to describe them) can be ordered, such that each is exactly one more than the previous value, and one less than the value after it.

Here we consider not only children's developing knowledge of the successor principle (for numbers less than 10), but also their knowledge of exact ordering. We consider each concept separately, and we also consider how these pieces of knowledge relate to each other in children's developing knowledge of natural number.

1.2. Relationship between cardinal principle and successor principle knowledge

As a group, children who understand the cardinal principle also typically perform above chance on a task tapping knowledge of the *successor principle* (Sarnecka & Carey, 2008). Nevertheless, little is known about *how* learning the cardinal principle relates to the acquisition of the concept of succession. One theoretical proposal posits that learning the cardinal principle entails learning the successor principle (Sarnecka & Carey, 2008). To explain the transition from four-knower to cardinal-principle-knower, Carey (2009) proposed a bootstrapping theory in which children learn the count list initially as a sequence of meaningless placeholders. They then learn the individual meanings of "one", "two", "three", and "four" by mapping these words onto the enriched parallel-individuation system in which long-term memory representations of particular sets are mapped onto verbal number words (Le Corre & Carey, 2007). These representations can then be used to create one-to-one correspondence with other small groups, such that, for example, any set that maps onto the representation in long-term memory representation of "Thing A-Thing B-Thing C" gets assigned the verbal label "three." Because the enriched parallel-individuation system has an upper limit of 4 items, children are unable to map the number words above "four" onto states of this system directly. Instead, Carey and colleagues theorized that children begin to notice the correspondence between the number words whose cardinal meanings they have learned individually ("one", "two", "three", and "four") and the order of the count list. Specifically, they argued that children notice that the next number in the count list corresponds to a state of the enriched parallel-individuation system that has one more item than the previous number in the count list. In other words, a four-knower would notice that "two" refers to sets that are exactly one more than "one", that "three" refers to sets that are exactly one more than "two, and that "four" refers to sets that are exactly one more than "three." The child then makes a logical induction that the successor principle holds for all numbers in their count list—in other words, that the next number in their count list refers to a set size exactly one more than the previous number in their count list. According to this theory, inducing the successor principle is what propels children to become cardinal-principle-knowers (CP-knowers).

To assess this theory, Carey and colleagues developed a task to measure children's understanding of the successor principle (Sarnecka & Carey, 2008). In this task, called the Unit task, the experimenter puts a certain number of objects into a box while labeling them (e.g., "I'm putting 5 buttons in the box"), adds either 1 or 2 objects to the box, and asks the child whether there are now 6 or 7 buttons in the box. A child who understands the successor principle should know that adding one object to a set of N (in this case, 5 buttons) means that the set now contains N + 1 (in this case, 6 buttons, the next number in the count list). Consistent with the bootstrapping theory, subset-knowers, including 3- and 4-knowers, performed at chance on this task, whereas CP-knowers performed above chance.

Although this result is consistent with the bootstrapping theory, it does not rule out the possibility that CP-knowers learn the successor principle *after* becoming CP-knowers, rather than *as* they learn the cardinal principle. Indeed, several studies have shown that children can understand the cardinal principle without succeeding on tasks that require understanding the successor principle (Davidson, Eng, & Barner, 2012; Wagner, Kimura, Cheung, & Barner, 2015). Many CP-knowers showed little or no evidence of understanding the successor principle, performing at or below chance on the Unit task. This is notable because the successor principle is implicitly represented in correct implementation of the counting procedure (i.e., counting that satisfies the counting principles of 1-to-1 correspondence, stable order, and the cardinal principle); however, this implicit representation is insufficient for CP-knowers to succeed on the Unit task, which requires applying successor principle knowledge in a different context than counting. In addition, among CP-knowers in these studies, performance on the Unit task was associated with performance on other numerical tasks, including counting fluency and the ability to estimate set sizes without counting, suggesting that the individual differences reflect true differences in number knowledge rather than artifacts of noise in the data. As a result, Davidson et al. (2012) have argued that children may *only* be able to learn succession once they grasp the cardinal principle.

Because the available studies have used only cross-sectional data, they have not been able to answer the question of whether learning the cardinal principle is *necessary* for children to learn the successor principle. That is, is the cardinal principle a "gatekeeper" skill that is needed before children can learn about succession? This question requires looking at change over time.

Two learning trajectories relating succession and cardinal principle knowledge are plausible, given current research. One possibility is that children learn the successor principle only *after* the cardinal principle, as Davidson and colleagues have argued. A second possibility is that children learn the successor principle and cardinal principle independently, and can learn the two concepts in either order. To distinguish between these possibilities, we need to compare learning over time among children who are not yet CP-knowers (3- and 4-knowers) to children who have learned the cardinal principle on the same successor principle tasks.

The two possible learning trajectories make divergent, testable predictions about who should improve over time in their understanding of the successor principle. Our prediction is that children *must* learn the cardinal principle prior to learning the successor principle, because it requires children to think not only about the cardinal meanings of number words, but also about the meanings of the words and values in relation to each other. If this is the case, then *only* children who are cardinal-principle-knowers should improve their understanding of the successor principle over time. Alternatively, if knowledge of the successor principle and cardinal principle develop independently, then cardinal principle knowledge should not relate to improvement on the