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Conceptual and procedural distinctions between fractions and decimals: A cross-national comparison

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ABSTRACT

Previous work has shown that adults in the United States process fractions and decimals in distinctly different ways, both in tasks requiring magnitude judgments and in tasks requiring mathematical reasoning. In particular, fractions and decimals are preferentially used to model discrete and continuous entities, respectively. The current study tested whether similar alignments between the format of rational numbers and quantitative ontology hold for Korean college students, who differ from American students in educational background, overall mathematical proficiency, language, and measurement conventions. A textbook analysis and the results of five experiments revealed that the alignments found in the United States were replicated in South Korea. The present study provides strong evidence for the existence of a natural alignment between entity type and the format of rational numbers. This alignment, and other processing differences between fractions and decimals, cannot be attributed to the specifics of education, language, and measurement units, which differ greatly between the United States and South Korea.

1. Introduction

1.1. Conceptual and processing differences between fractions and decimals

A major conceptual leap in the acquisition of formal mathematics takes place with the introduction of rational numbers (typically fractions followed by decimals, at least in curricula used in the United States). These are the first formal numbers students encounter that can represent magnitudes less than one. Both fraction and decimal symbolic notations often prove problematic for students. Children, and even some adults, exhibit misconceptions about the complex conceptual structure of fractions (Ni & Zhou, 2005; Siegler, Fazio, Bailey, & Zhou, 2013; Siegler, Thompson, & Schneider, 2011; Stigler, Givvin, & Thompson, 2010). Such difficulties have also been reported in high mathematics-achieving countries such as South Korea (Kim & Whang, 2011, 2012; Kwon, 2003; Pang & Li, 2008). Students also encounter problems in learning to understand decimals (Rittle-Johnson, Siegler, & Alibali, 2001), but generally master the magnitudes of decimals before fractions (Iuculano & Butterworth, 2011).

Fractions and decimals are typically introduced as alternative notations for the same magnitude, other than rounding error (e.g., 3/8 km vs. 0.375 km). For example, the Common Core State Standards Initiative (2014) for Grade 4 refers to decimals as a “notation for fractions”. However, psychological research has revealed both conceptual and processing differences between the two notations. Whereas the bipartite (a/b) structure of a fraction represents a two-dimensional relation, a corresponding decimal represents a one-dimensional magnitude (English & Halford, 1995; Halford, Wilson, & Phillips, 1998) in which the variable denominator of a fraction has been replaced by an implicit constant (base 10). Studies have shown that magnitude comparisons can be made much more quickly and accurately with decimals than with fractions (DeWolf, Grounds, Bassok, & Holyoak, 2014; Iuculano & Butterworth, 2011), but that fractions are more effective than decimals in tasks such as relation identification or analogical reasoning, for which relational information is paramount (DeWolf, Bassok, & Holyoak, 2015a). Importantly, various aspects of performance with both fractions and decimals predict subsequent success with more advanced mathematical topics, such as algebra (Booth, Newton, & Twiss-Garrity, 2014; DeWolf, Bassok, & Holyoak, 2015b; Siegler et al., 2011, 2012, 2013).

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1.2. Semantic alignment and the ontology of quantity types

There is considerable evidence that people's interpretation and use of arithmetic operations is guided by *semantic alignment* between mathematical and real-life situations. The entities in a problem situation evoke semantic relations (e.g., tulips and vases evoke the functionally asymmetric "contain" relation), which people align with analogous mathematical relations (e.g., the noncommutative division operation, tulips/vases) (Bassok, Chase, & Martin, 1998; Guthormsen et al., 2015). Rapp, Bassok, DeWolf, and Holyoak (2015) found that a form of semantic alignment guides the use of different formats for rational numbers, fractions and decimals. Specifically, adults in the United States selectively use fractions and decimals to model discrete (i.e., countable) and continuous entities, respectively. Similarly, DeWolf et al. (2015a) demonstrated that American college students prefer to use fractions to represent ratio relations between countable sets, and decimals to represent ratio relations between continuous quantities.

The preferential alignment of fractions with discrete quantities and decimals with continuous quantities appears to reflect a basic ontological distinction among quantity types (e.g., Cordes & Gelman, 2005). Sets of discrete objects (e.g., the number of girls in a group of children) invite counting, whereas continuous mass quantities (e.g., height of water in a beaker) invite measurement. Continuous quantities can be subdivided into equal-sized units (i.e., discretized) to render them measurable by counting (e.g., slices of pizza), but the divisions are arbitrary in the sense that they do not isolate conceptual parts. Even for adults, the distinction between continuous and discrete quantities has a strong impact on selection and transfer of mathematical procedures (Alibali, Bassok, Olseth, Syc, & Goldin-Meadow, 1999; Bassok & Holyoak, 1989; Bassok & Olseth, 1995).

The different symbolic notations for rational numbers, fractions and decimals, appear to have different natural alignments with discrete and continuous quantities (see Fig. 1). A fraction represents the ratio formed between the cardinalities of two sets, each expressed as an integer; its bipartite format (a/b) captures the value of the part (the numerator a) and the whole (the denominator b). A decimal can represent the one-dimensional magnitude of a fraction ($a/b = c$) expressed in the standard base-10 metric system.

The fraction format is well-suited for representing sets and subsets of discrete entities (e.g., balls, children) that can be counted and aligned with the values of the numerator (a) and the denominator (b) (e.g., 3/7 of the balls are red). Also, as is the case with integer representations, the fraction format can be readily used to represent continuous entities that have been discretized—parsed into distinct equal-size units—and therefore can be counted (e.g., 5/8 of a pizza). In contrast, the one-dimensional decimal representation of such discrete or discretized entities seems much less natural (~0.429 of the balls are red; 0.625 of a pizza).

In contrast, the decimal format is well-suited to represent portions of continuous entities, particularly since unbounded decimals capture all real numbers (i.e., all points on a number line). This alignment appears to be especially strong when decimals (base 10) are used to model entities that have corresponding metric units (0.3 m, 0.72 l). When continuous entities have non-metric units (e.g., imperial measures with varied bases such as 12 in. or 60 min), their alignment with decimals may require computational transformations. Given that the denominator of a fraction is a variable that can be readily adapted to any unit base, it is computationally easier to represent non-metric measures of continuous entities with fractions (2/3 of a foot) than with decimals (0.67 ft). Because computational ease likely interacts with the natural conceptual alignment of continuous entities with decimals, metric units are predominantly represented with decimals, whereas imperial units may be represented by fractions (Rapp et al., 2015).

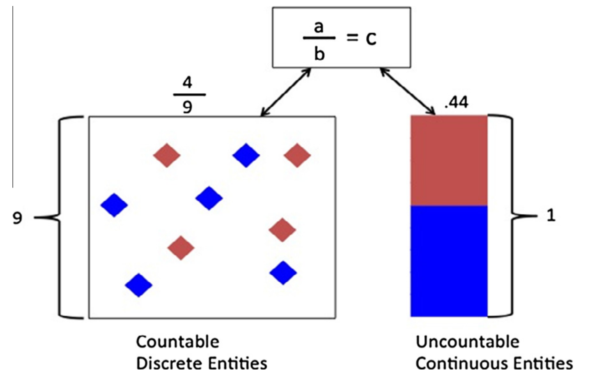

Fig. 1. Hypothesized alignment of fractions and decimals with discrete and continuous entities. Copyright © 2015 by the American Psychological Association. Reproduced with permission from Rapp et al. (2015).

1.3. The need for cross-national comparisons

The conceptual and processing differences between the different notations for rational numbers have been interpreted as reflecting basic representational differences between alternative formats for such numbers. Fractions may be better suited to represent two-dimensional relations (DeWolf et al., 2015a), whereas decimals may be more closely linked to one-dimensional magnitude values (DeWolf et al., 2014). In addition, the mental representations of fractions and decimals may inherently align with discrete and continuous quantities, respectively (Rapp et al., 2015).

However, the interpretation of these findings as reflections of deep representational distinctions remains speculative, as all the phenomena we have reviewed have been demonstrated only with American students. It is well-known that students in the United States lag behind students in various Asian countries (including South Korea, Singapore, and Japan) in their math achievement (OECD, 2012). Perhaps the gaps observed between performance on various tasks (e.g., the superiority of decimals in magnitude comparison, or of fractions in relational reasoning) reflect deficiencies in the knowledge American students have attained about rational numbers. Similarly, the distinction between discrete and continuous entities has linguistic and cultural correlates (Geary, 1995); hence it is possible that non-English-speaking students from a different culture would not align distinct mathematical symbols with distinct types of quantity. Such interpretive issues can be addressed by cross-national and cross-cultural research (cf. Bailey et al., 2015; Hiebert et al., 2003; Richland, Zur, & Holyoak, 2007; Stigler, Fernandez, & Yoshida, 1996). In order to develop general theories in the field of higher cognition, it is critical to distinguish between phenomena that are specific to particular educational practices in specific contexts, and those that reflect representational capacities of the human mind that are not determined by specific educational practices or cultural contexts. The methodological approach of identifying those aspects of cognitive performance that are the same or different across populations varying in culture, language, and educational practices is especially informative in answering these types of basic questions.

1.4. Overview of the present study

Here we report a cross-national comparison of conceptual and processing differences between fractions and decimals. We systematically replicated several studies conducted in the United States that compared performance with the two types of rational numbers, using tasks involving both magnitude comparison and relational reasoning, with samples drawn from college students in South Korea. Several factors make South Korea a particularly