Cognition 146 (2016) 48–57

Contents lists available at ScienceDirect

Cognition

journal homepage: www.elsevier.com/locate/COGNIT

Ten-year-old children strategies in mental addition: A counting model account

Catherine Thevenot *, Pierre Barrouillet, Caroline Castel, Kim Uittenhove

University of Geneva, FAPSE, Switzerland

ARTICLE INFO

Article history:
Received 13 June 2014
Revised 21 July 2015
Accepted 6 September 2015

Keywords:
Numerical cognition
Learning process
Mental arithmetic
Mathematical cognition

ABSTRACT

For more than 30 years, it has been admitted that individuals from the age of 10 mainly retrieve the answer of simple additions from long-term memory, at least when the sum does not exceed 10. Nevertheless, recent studies challenge this assumption and suggest that expert adults use fast, compacted and unconscious procedures in order to solve very simple problems such as 3 + 2. If this is true, automated procedures should be rooted in earlier strategies and therefore observable in their non-compacted form in children. Thus, contrary to the dominant theoretical position, children's behaviors should not reflect retrieval. This is precisely what we observed in analyzing the responses times of a sample of 42 10-year-old children who solved additions with operands from 1 to 9. Our results converge towards the conclusion that 10-year-old children still use counting procedures in order to solve non-tie problems involving operands from 2 to 4. Moreover, these counting procedures are revealed whatever the expertise of children, who differ only in their speed of execution. Therefore and contrary to the dominant position in the literature according to which children's strategies evolve from counting to retrieval, the key change in development of mental addition solving appears to be a shift from slow to quick counting procedures.

1. Introduction

Among numerical activities, mental addition has been one of the arithmetic tasks most studied by cognitive psychology. For more than 40 years, it has been admitted that, while young children aged 6 or 7 use counting procedures, adults after repetitive practice do not have to count any longer in order to solve simple addition problems such as 3 + 2. Instead, they would quickly retrieve the answers of such problems from a network stored in long-term memory (e.g., Ashcraft & Battaglia, 1978; Groen & Parkman, 1972 for seminal studies). The developmental pattern from counting to retrieval is commonly described as follow. Because addition is not commonly learnt by rote at school, young children start solving simple problems through the use of external aids such as objects or fingers. The concrete counting-all procedure (i.e., CCA strategy, Fuson, 1982) is the most primitive strategy where children represent the first addend with objects or fingers, do the same for the second addend and then count all the objects. Then, children use increasingly sophisticated strategies (see Baroody & Tiilikainen, 2003, for a complete description) and eventually count-on from the larger addend without resort to concrete objects (i.e., COL strategy, Fuson, 1982; MIN strategy, Groen & Parkman, 1972). Improvement in children's conceptual knowledge of counting is reflected in a gradual shift from the use of basic strategies towards the more efficient Min strategy (Geary, Bow-Thomas, & Yao, 1992; Geary & Hoard, 2002; Siegler, 1987). The use of counting procedures would eventually result in the construction of memory representations of basic facts (Ashcraft, 1992; Ashcraft & Fierman, 1982; Geary, Brown, & Samaranayake, 1991; Siegler & Shrager, 1984). Indeed, the repetitive co-occurrence of the problem operands and the answer in working memory would lead to the storage of associations between the three numbers (Logan, 1988). Therefore, chunks of knowledge taking the form of arithmetic facts could ultimately be retrieved from long term memory in order to quickly produce problem answers (Siegler & Shipley, 1995; Siegler & Shrager, 1984).

However, we have recently questioned this large consensus and suggested that, on the contrary, expert adults would still use counting procedures in order to solve very simple addition such as 3 + 2. This provocative conclusion was first formulated after we showed that problem solving is facilitated when the arithmetic sign is presented 150 ms before the operands for simple additions but not for multiplications (Fayol & Thevenot, 2012). We inferred from these results that abstract procedures were primed by the

* Corresponding author at: FPSE, University of Geneva, 40, bd du Pont D'Arve, CH-1205 Geneva, Switzerland.

E-mail address: catherine.thevenot@unige.ch (C. Thevenot).

http://dx.doi.org/10.1016/j.cognition.2015.09.003
0010-0277/

"+" sign and consequently used to solve addition problems. For multiplication, presenting the "×" sign before the operands did not pre-activate procedures, hence the lack of facilitation effect. The conclusion that procedures are still used by adults for addition was reinforced in a second study wherein we measured adult solution times to additive problems involving operands from 1 to 4 (Barrouillet & Thevenot, 2013). The results revealed that response times monotonically and linearly increased by about 20 ms each time either the augend or the addend was incremented by one. The systematic size effect resulting in this linear pattern also strongly suggests the use of counting procedures. Therefore, our results converge towards the assumption that adults resort to extremely fast procedures to solve simple additions.[1] The fact that adults massively report retrieval for problems such as 3 + 2 or 4 + 3 (LeFevre, Sadesky, & Bisanz, 1996) could suggest that these procedures have been automatized and are no longer under conscious control. In other words, only the outcome of the process is accessible and not any longer the process in itself (Newell, 1990). The object of the present study is to determine how these arithmetic procedures have evolved until automatization and unconsciousness.

Our conclusion that adults use rapid procedures when they solve simple additions necessarily challenges the current developmental conception that efficiency in arithmetic increases through a shift from counting to retrieval. Instead, efficiency would increase following an acceleration of early counting procedures (Baroody, 1984, 1994; Baroody & Varma, 2006). Effortful count-all and count-on strategies would progressively be compiled and the cognitive demand of initially fastidious step by step counting procedures would therefore progressively diminish with practice. If we are right in assuming that counting procedures used by the fully-developed mind are rooted in basic counting strategies and are compiled until their automatization, we should be able to track this mechanism through cognitive development. Indeed, if there is no strategy shift across aging, schooling or expertise acquisition, the behavioral patterns of less and more efficient children's as well as of adult should remain the same across development and only an acceleration of procedures should be observed. In other words, the scale wherein solution times are represented should shrink with practice without major changes in solution time distributions. On the contrary, if there is a modification in the nature of strategies during development, the shift from counting to retrieval should be revealed by a rupture in individual's behavior. Then, solution time distributions should not follow the same pattern depending on individuals' expertise.

Thus, we predict that children will exhibit the same overall pattern as the one observed by Barrouillet and Thevenot (2013) in adults, with a linear increase in response times with the size of both operands. Note, however, that such size effects do not necessarily provide direct evidence for counting procedures. Because of frequency variations in problem occurrence, a size effect can also be accounted for by retrieval models. Memory access could be easier for smaller problems because they are encountered earlier in development and more often solved than larger problems (Ashcraft & Christy, 1995; Hamann & Ashcraft, 1985). As a consequence, memory traces of small problems would be more vivid and quicker to access, hence variations in retrieval times. Therefore, both retrieval and counting models could account for variations in solution times but this is not to say that observations of solution time distributions are not useful for the identification of strategy used. Indeed, the magnitude of the size effect can help us in distinguishing retrieval from counting. More concretely, retrieval rate variations on frequent small additions involving operands from 1 to 4 are necessarily relatively limited compared to variations stemming from step by step procedures. For example, variations in response times of several hundreds of ms per increment can most probably be imputed to a counting procedure rather than to variations in efficiency of some process of direct retrieval from memory. In the same way, problem size effects within retrieval models have sometimes been attributed to individuals' sporadic use of counting (e.g., Groen & Parkman, 1972; LeFevre et al., 1996). However, such a hypothesis is only compatible with problem-size effects of moderate magnitude (i.e., some tens of ms. in Groen & Parkman). The variation in response times caused by the use of slow and effortful counting strategies we hypothesize in children should be far beyond these restricted limits.

In the present study, the prediction of a linear increase in response times with the size of both operands will be tested through analyses of variance on the mean response times with the size of both operands as independent variables. However, in order to provide a complete description of children's behaviors, we will also examine through regression analyses the impact on response times of different predictors that are traditionally taken into account in the literature. The product of the two operands, their sum, the square of this sum, the sum of each operand squared, as well as the minimum and maximum addend will be entered in our analyses. We predict a linear organization of solution times. They should increase as a function of the sum of the operands, as we observed in adults (Barrouillet & Thevenot, 2013). Moreover, as described above, amongst counting procedures, the Min strategy is the more likely to be used by proficient children and, consequently, the minimum addend should operate as a very good predictor of solution time variations. By contrast, retrieval models would predict that children's solution times will not follow a linear but an exponential trend with the squared sum of the operands as the best predictor (Ashcraft, 1982, 1992; Ashcraft & Battaglia, 1978; Ashcraft & Fierman, 1982). The predictive power of the squared sum was thought to be incompatible with counting models and rather compatible with the view that retrieval of the problem answers was performed by searching a square tabular memory network stretched in the direction of larger sum (Ashcraft & Battaglia, 1978). Entering the table at its origin (0, 0), the search would progress towards the relevant column, scrolling it until reaching the intersection with the appropriate line. Unfortunately, Widaman, Geary, Cormier, and Little (1989) demonstrated that Ashcraft erred in the conception of his model, which should, in fact, be associated with the sum of each squared operand, and not their squared sum, as the best predictor. In addition to this first criticism, Widaman et al. noted that Ashcraft's city block-metric conception does not really fit with cognitive network modelization and that spreading models better suit our current knowledge of human brain organization (Anderson, 1983; Collins & Loftus, 1975). Within this conception, the product of the operands would be the best solution time predictor. Indeed, the answer of the problem would be retrieved following a spreading activation within a rectangle whose length and width correspond to the size of the first and second operands. Consequently, the area to be filled up until the answer node is reached is equal to the product of the operands (Widaman et al., 1989).

In order to examine these possibilities, we studied 10-year-old children in fourth grade and asked them to solve very simple addition problems involving operands from 1 to 4. This specific age group was chosen because retrieval is already supposed to be the

[1] Note that our conclusions have been recently questioned by Campbell and Beech (2014) who showed generalization of practice for $N + 0$ problems but not for any other simple addition problems. The authors concluded that only addition problems involving 0 are solved through a generalizable procedure whereas this lack of generalization is the indicator of retrieval for the other problems. Nevertheless, Campbell and Beech's conclusions have, in turn, been challenged by Baroody, Eiland, Purpura, and Reid (2014) who noted that generalization effects should have also been observed for addition problems involving 1, which are known to be solved by procedural rules.