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Running the number line: Rapid shifts of attention in single-digit arithmetic

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ABSTRACT

It has been recently proposed that adults might solve single-digit addition and subtraction problems by rapidly moving through an ordered representation of numbers. In the present study, we tested whether these movements manifest themselves by on-line shifts of attention during arithmetic problem-solving. In two experiments, adult participants were presented with single-digit addition, subtraction and multiplication problems. Operands and operator were presented sequentially on the screen. Although both the first operand and the operator were presented at the center of the screen, the second operand was presented either to the left or to the right side of space. We found that addition problems were solved faster when the second operand appeared to the right than to the left side (Experiments 1 & 2). In contrast, subtraction problems were solved faster when the second operand appeared to the left than to the right side (Experiment 1). No operation-dependent spatial bias was observed in the same time window when the second operand was zero (Experiment 1), and no bias was observed when the operation was a multiplication (Experiment 2). Therefore, our results demonstrate that solving single-digit addition and subtraction, but not multiplication, is associated with horizontal shifts of attention. Our findings support the idea that mental movements to the left or right of a sequential representation of numbers are elicited during single-digit arithmetic.

1. Introduction

Mastering basic arithmetic is a major goal of elementary education and an essential first step toward higher-level mathematical abilities. Therefore, the strategies used by skilled adults to solve simple arithmetic problems have been the focus of a large body of literature over the past 40 years (Ashcraft and Guillaume, 2009, for a recent review). Using verbal reports and chronometric data, studies have converged to indicate that answers of simple arithmetic problems (such as single-digit addition, subtraction and multiplication) can either be retrieved from long-term memory (Campbell & Xue, 2001; Geary, Frensch, & Wiley, 1993; LeFevre, Sadesky, & Bisanz, 1996) or calculated using algorithmic procedures (e.g., counting, decomposition) (Barrouillet, Mignon, & Thevenot, 2008; Cooney, Swanson, & Ladd, 1988; Robinson, 2001; Seyler, Kirk, & Ashcraft, 2003). Typically, algorithmic procedures are seen as slow and effortful, whereas direct retrieval is considered fast and efficient. Therefore, there is a relative consensus in the literature that effective arithmetic learning is characterized by a shift from procedural to retrieval strategies (Ashcraft, 1982, 1992; Ashcraft & Guillaume, 2009; Geary, 1994; Siegler, 1996; Siegler & Shrager, 1984). In other words, the repetitive co-occurrence of a given problem with its answer during childhood would lead to a progressive association between that particular problem and answer in long-term memory (Geary & Burlingham-Dubree, 1989; Logan, 1988; Siegler & Shipley, 1995). The result is that skilled adults would not recruit procedural knowledge but largely rely on direct retrieval when solving simple arithmetic problems (Campbell & Xue, 2001; Geary et al., 1993). Algorithmic procedures would be mostly engaged when solving less practiced problems for which there is weak association between operands and answer (e.g., large problems) (LeFevre et al., 1996; Núñez-Peña, Gracia-Bafalluy, & Tubau, 2011; Thevenot, Barrouillet, & Fayol, 2001; Thevenot, Fanget, & Fayol, 2007).

Recently, a study cast doubt on this consensus. Using a priming paradigm, Fayol and Thevenot (2012) showed that skilled adults were faster at solving even very simple addition and subtraction

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problems (e.g., 3 + 2, 3 − 2) when the operation sign was presented 150 ms prior to the operands than when it was presented at the same time (see also Roussel, Fayol, & Barrouillet, 2002). Because no such priming was observed for single-digit multiplication, the effect appears to be operation-specific and may reflect the pre-activation of fast and automated procedures that could subsequently be used to solve addition and subtraction (but not multiplication) problems. Therefore, unlike what has been widely assumed in the past decades, procedural knowledge may still be recruited for solving even very simple addition and subtraction problems in skilled adults. Such procedural knowledge might not be recruited when solving multiplication problems, most likely because associations between operands and answers are explicitly learned by rote in school and only retrieved from long-term memory (Dehaene & Cohen, 1995).

A fundamental question arising from the findings of Fayol and Thevenot (2012) concerns the nature of the automated procedures that would be associated with single-digit addition and subtraction. It has been recently proposed that such procedures could take the form of a "process of rapid scrolling through an easily accessible and overlearned representation stored in long-term memory" (Barrouillet & Thevenot, 2013, p. 43). This proposal is consistent with the fact that solution times of even very small addition problems linearly increase as a function of operand size in adults (i.e., solution time increases with the distance between the original value and the value corresponding to the sum) (Barrouillet & Thevenot, 2013; Groen & Parkman, 1972). It suggests that the step-by-step counting procedures used by children when learning arithmetic might not totally disappear but instead be replaced by automatized counting procedures in adults (Barrouillet & Thevenot, 2013; Fayol & Thevenot, 2012). More generally, this proposal harks back to the idea that a key change in acquiring arithmetic efficiency may involve a shift from slow informal counting procedures to compiled procedural knowledge (Baroody, 1983, 1984, 1994). Because such internalized procedures do not necessarily reach consciousness, it could lead participants to mistakenly report using retrieval (Anderson, 1983; Ric & Muller, 2012).

If solving simple addition and subtraction problems does indeed involve rapid movement along an ordered representation of numbers, there are good reasons to assume that this process and representation are spatial in nature. Indeed, a growing number of studies document a link between numerical cognition and space (for a recent review, see Fischer & Shaki, 2014a). For example, studies have found that numbers are associated with a spatial bias in manual responses: When participants compare the magnitude of numbers (or classify them as even or odd), small numbers are processed faster with the left hand than with the right hand whereas large numbers are processed faster with the right hand than with the left hand (Dehaene, Bossini, & Giraux, 1993; Wood, Willmes, Nuerk, & Fischer, 2008). Numbers also automatically induce spatial shifts of attention. Specifically, small numbers facilitate the detection of a subsequent target in the left half of visual field (hereafter referred to as left hemifield) while large numbers facilitate the detection of a subsequent target in the right half of visual field (hereafter referred to as right hemifield) (Fischer, Castel, Dodd, & Pratt, 2003). Overall, these effects indicate that participants may represent numbers as spatially ordered items along a mental number line (MNL), with smaller magnitudes on the left side and larger magnitudes on the right side (Dehaene et al., 1993; Hubbard, Piazza, Pinel, & Dehaene, 2005).

More recently, studies have suggested that such spatial biases are not restricted to numbers but might also be present in symbolic arithmetic (Fischer & Shaki, 2014a, 2014b). Most evidence for a link between symbolic arithmetic and space comes from studies on complex arithmetic (i.e., problems involving multi-digit numbers that are typically not thought to be retrieved from memory). For example, Knops, Viarouge, and Dehaene (2009) showed that adults generally overestimate the result of complex symbolic addition while they underestimate the results of complex symbolic subtraction, an effect called *operational momentum* (OM) effect. As suggested by some (Hubbard et al., 2005; Knops, Dehaene, Berteletti, & Zorzi, 2014; Knops, Zitzmann, & McCrink, 2013; Knops et al., 2009), the OM effect might indicate that participants solve addition and subtraction problems by shifting their attention rightward or leftward along the MNL. The OM effect might stem from the fact that participants might move "too far" to the right (or to the left) along the MNL when solving an addition (or a subtraction) problem, leading to an overestimation (or an underestimation) of the actual result. Two studies provide further support for this attentional shift hypothesis. First, using functional magnetic resonance imaging (fMRI), Knops, Thirion, Hubbard, Michel, and Dehaene (2009) showed that multi-digit addition and subtraction problems are associated with different patterns of brain activation in the posterior superior parietal lobule (PSPL), a region involved in visuo-spatial processing. They further showed that the pattern of brain activation associated with multi-digit addition in that region is similar to the pattern of activation associated with rightward saccades (in line with the idea that participants shift their attention to the right of the MNL when solving multi-digit addition problems). Second, Klein, Huber, Nuerk, and Möller (2014) recorded eye movements of participants while they had to locate the results of (predominantly) multi-digit addition and subtraction results on a given number line. Consistent with the attentional shift hypothesis, the authors found that participants moved their eyes to the right of their first fixation on the line when they located the results of addition problems, while they moved their eyes to the left of their first fixation when they located the results of subtraction problems. Overall, these studies suggest that the procedures used by skilled adults to solve complex arithmetic problems might involve mentally moving along a spatial MNL.

The idea that addition and subtraction would involve asymmetric shifts of attention along the MNL is broadly consistent with Barrouillet and Thevenot's proposal of moving along a representation of numbers (Barrouillet & Thevenot, 2013). Yet, it remains unknown whether these attentional shifts are elicited on-line during the resolution of simple arithmetic problems (i.e., problems involving single-digit numbers that are typically thought to be retrieved) and could provide the basis for the fast and automatic procedures hypothesized by Barrouillet and Thevenot (2013) and Fayol and Thevenot (2012). To our knowledge, only a few studies have investigated the link between space and simple arithmetic problem-solving.

First, Pinhas and Fischer (2008) asked participants to point to the results of single-digit arithmetic problems on a number line that was visually presented. For a same result (e.g., "6"), participants' pointing was biased to the right for an addition (e.g., 4 + 2) and to the left for a subtraction (e.g., 8 − 2). Therefore, this study indicates the presence of an OM in simple arithmetic. However, problems containing zero were associated with an even larger OM than other problems. This is inconsistent with the idea that the OM in that study stems from shifts of attention elicited by arithmetic calculation because addition and subtraction problems containing zero should not require any differential movement along the MNL. Thus, the authors proposed a "spatial competition account" according to which each component of an arithmetic problem (i.e., the operands, the operator and the result) leads to competing spatial activations along the MNL (Pinhas & Fischer, 2008). Another account posits that the OM might be accounted for by the heuristic "accepting more than the first operand" for addition and "accepting less than the first operand" for subtraction (Knops et al., 2009; McCrink & Wynn, 2009). Other alternative accounts that do not involve shifts along the MNL have been proposed