Cognition 146 (2016) 289–303

Contents lists available at ScienceDirect

Cognition

journal homepage: www.elsevier.com/locate/COGNIT

Fast automated counting procedures in addition problem solving: When are they used and why are they mistaken for retrieval? ☆

Kim Uittenhove, Catherine Thevenot, Pierre Barrouillet *

Université de Genève, Switzerland

A R T I C L E I N F O

Article history:
Received 4 November 2014
Revised 14 September 2015
Accepted 11 October 2015

Keywords:
Problem solving
Retrieval
Arithmetic
Association
Numerical cognition

A B S T R A C T

Contrary to a widespread assumption, a recent study suggested that adults do not solve very small additions by directly retrieving their answer from memory, but rely instead on highly automated and fast counting procedures (Barrouillet & Thevenot, 2013). The aim of the present study was to test the hypothesis that these automated compiled procedures are restricted to small quantities that do not exceed the size of the focus of attention (i.e., 4 elements). For this purpose, we analyzed the response times of ninety adult participants when solving the 81 additions with operands from 1 to 9. Even when focusing on small problems (i.e. with sums $\leqslant$10) reported by participants as being solved by direct retrieval, chronometric analyses revealed a strong size effect. Response times increased linearly with the magnitude of the operands testifying for the involvement of a sequential multistep procedure. However, this size effect was restricted to the problems involving operands from 1 to 4, whereas the pattern of response times for other small problems was compatible with a retrieval hypothesis. These findings suggest that very fast responses routinely interpreted as reflecting direct retrieval of the answer from memory actually subsume compiled automated procedures that are faster than retrieval and deliver their answer while the subject remains unaware of their process, mistaking them for direct retrieval from long-term memory.

1. Introduction

The associative nature of memory is the object of a large consensus in cognitive psychology. As Anderson (1974) noted, the idea that objects or thoughts that have been experienced in close contiguity become associated in memory (Thorndike, 1922), and that these associations govern the subsequent recollection of these objects or thoughts can be traced back to Aristotle in his essay “On memory and reminiscence”. Nonetheless, modern theories went further than Aristotle’s insights and no longer view memory as a muddled depository of imprints left by experienced contiguities, but as hierarchically structured systems that store organized bundles of associations (e.g., Anderson, 1974; Anderson, 1993; Collins & Quillian, 1972). These neo-associationist theories also suppose that associations can bind together elements that are not necessarily perceived, but also produced by mental computation (Anderson, 1993). The recurrent solving of a problem is assumed to lead to the association in memory of this problem with its answer, an associative process seen as highly adaptive because it is assumed that directly retrieving answers from memory would provide us with faster and more accurate responses than any algorithmic reconstructive process (Logan, 1988).

This theoretical framework has found one of its most perfect fields of application in the domain of mental arithmetic and simple addition problem solving. Before any systematic tuition in primary school, children develop a variety of counting strategies for solving simple additions. These strategies that initially rely on manipulatives (objects or fingers) become rapidly internalized as verbal counting. Eventually, solving frequently encountered problems by counting procedures leads to their association in long-term memory with the computed answers, adult performance being characterized by the subsequent retrieval of these problem-answer associations. Consequently, development would take the form of a progressive shift from algorithmic problem solving to direct retrieval. The aim of this article is to put this conventional wisdom of cognitive psychology under scrutiny.

1.1. Retrieval of associations in mental arithmetic

A popular application of the associationist framework outlined above is probably the distribution of associations model proposed by Siegler and Shrager (1984). The model distinguishes between

☆ This research was supported by a grant from the Swiss National Science Foundation N° 100014_140257/1 to Pierre Barrouillet and Catherine Thevenot.

* Corresponding author at: Université de Genève, Faculté de Psychologie et des Sciences de l’Education, 40 bd du Pont d’Arve, 1211 Genève 4, Switzerland.

E-mail address: Pierre.Barrouillet@unige.ch (P. Barrouillet).

http://dx.doi.org/10.1016/j.cognition.2015.10.008
0010-0277/

the representation of knowledge about particular problems and strategies that operate on this knowledge to produce responses that in turn modify representations. These representations are conceived as associations of various strength between problems (e.g., 5 + 3) and potential answers that can be correct but also incorrect (e.g., 6, 7, 8, or 9). The determinant dimension of the strategy choice is the *peakedness* of the distribution of associations for a given problem. Some problems have a peaked distribution with an answer, ordinarily the correct answer, that concentrates almost all the associative strength. Other problems have a relatively flat distribution in which the associative strength is distributed among several answers. Retrieving a given problem-answer association within this model depends on three parameters: its relative strength over all the other associations, a confidence criterion that determines the associative strength that must be exceeded for successful retrieval, and a search length criterion that determines the number of retrieval efforts the subject will make before moving to another strategy. The problem is solved through retrieval if an answer is found with an associative strength that exceeds the confidence criterion before reaching the search length deadline. As a consequence, retrieval is more probable for problems with a peaked than a flat distribution.

More relevant for the present study is the assumption of the authors about how children acquire these distributions. In line with the associationist framework, Siegler and Shrager (1984) assume that each time children answer a problem, the associative strength linking this problem to that answer increases, whatever this answer and the strategy used. Thus the probability of retrieval is influenced by the frequency of exposure to the problem, which determines the opportunities to learn answers, and the sum of the two addends, with a greater probability to err when using counting procedures on large numbers. A computational simulation of the model integrating these factors showed that the choice of strategy converges toward direct retrieval, especially for the smallest problems that are more frequently and accurately solved by preschoolers.

This model has received strong support from several studies (Barrouillet & Fayol, 1998; Campbell & Timm, 2000; Geary & Brown, 1991; Geary & Burlingham-Dubree, 1989; Hamann & Ashcraft, 1986; Imbo & Vandierendonck, 2007; Imbo & Vandierendonck, 2008; Reder, 1988) and has provided a theoretical basis to the recurrent observation that adults retrieve from memory the answer of small additions instead of having to calculate it (Ashcraft, 1982; Ashcraft, 1987; Ashcraft & Battaglia, 1978; Ashcraft & Stazyk, 1981; Barrouillet & Fayol, 1998; Campbell, 1987a; Campbell, 1987b; LeFevre, Sadesky, & Bisanz, 1996; Miller, Perlmutter, & Keating, 1984). Thus, it is almost universally admitted that small additions have so often been encountered that their answer is necessarily retrieved from memory in adults (see Zbrodoff & Logan, 2005, for a review).

1.2. A discordant phenomenon: the problem-size effect

A straightforward prediction of the algorithmic computing/direct retrieval transition model would be the progressive attenuation and, at the end, the disappearance of the effects related with factors that affect performance when problems are solved through algorithmic computing. This is the case of the size of the operands in addition solving. In a seminal study, Groen and Parkman (1972) observed that the best predictor of the RTs in first graders asked to solve small additions (the largest problem was 5 + 4) was the size of the smaller of the two addends. This finding suggested the use of a counting procedure by which children start from the larger addend and then count on by ones for the value of the smaller addend (e.g., for 2 + 4, counting 4, 5, 6, a procedure known as the *Min* strategy). The observed slope of 410 ms per increment lent strong support to this hypothesis. Interestingly, tie problems (e.g., 3 + 3) seemed to remain immune to this problem-size effect. Characterized by smaller RTs than the other problems, they were assumed to be solved by direct retrieval of their answer from long-term memory, an idea that is now universally admitted. Groen and Parkman also investigated addition solving in adults. The hypothesis of a transition from algorithmic computing to direct retrieval would have predicted a generalization of the pattern observed in tie problems to all the small problems that were presented in the children study. However, the authors observed a small but significant slope of 20 ms associated with the size of the *Min*. Groen and Parkman judged these 20 ms an implausibly fast rate for a counting procedure, and suggested that adults solve small additions through retrieval, the remaining small size effect being due to the sporadic use of slower counting strategies in rare trials on which the retrieval strategy failed (approximately 5%).

This problem-size effect (i.e., the increase in latencies with the size of the *Min* or the sum of the two operands) has been observed in virtually all the studies, Zbrodoff and Logan (2005) entitling their review on this phenomenon "What everyone finds". The hypothesis of a size effect due to the use of slower non-retrieval strategies in some trials was buttressed by LeFevre et al. (1996) who observed that adults reported using retrieval in more than 80% of the small additions (sum $\leqslant$ 10), but in only 47% of the large additions (10 < sum $\leqslant$ 17) when ties were excluded. However, they also noted that RTs increased with problem size even in those trials that were reported as retrieved. This latter problem-size effect on retrieved small problems was reduced when compared with the effect on all the trials, but somewhat incompatible with the reported process of retrieval. It has nonetheless received several explanations. LeFevre et al. suggested that retrieval latencies could reflect acquisition history, with problems often solved through algorithmic strategies in the course of development resulting in flatter distributions of associations and longer retrieval latencies (e.g., Siegler & Shrager, 1984, contrasted the peaked distribution of 4 + 1 with the flatter distribution of 4 + 5). In the same way, Hamann and Ashcraft (1986, see also Ashcraft & Guillaume, 2009), suggested a memory strength model assuming that the frequency with which additive problems are practiced by children decreases as the size of the operands increases, leading to weaker associations (recall that the frequency of exposure to problems was one of the factors determining the probability of retrieval in the distribution of associations model). Along with this frequency hypothesis, structural properties of the problems have also been advocated. Ashcraft and Battaglia (1978) and Ashcraft and Stazyk (1981), who rejected Groen and Parkman's (1972) hypothesis of a size effect due to the sporadic recourse to slower non-retrieval strategies, suggested that it resulted from the time-course of a search through a tabular representation of the 100 basic addition facts. Beginning at 0,0 and progressing outward along the rows and columns until the intersection is reached, this search would take longer for larger operands. Because the best predictor of response times was the square of the sum in Ashcraft and Battaglia (1978), they hypothesized some stretching of the table in the region of the larger numbers resulting in a slowing down of the search process with larger operands. By contrast, Widaman, Geary, Cormier, and Little (1989), who found that the product of the two addends was the best predictor, hypothesized an equal spacing of the rows and columns of the table from 0 to 9. Assuming a process of spreading activation through the memory network, the time needed to reach a given intersection (i.e., the correct sum) would be proportional to the area of the network to be traversed, hence the predictive power of the product of the two addends. Zbrodoff (1995) and Zbrodoff and Logan (2005) proposed a network interference model in which problem-answer associations take longer to retrieve for larger problems because