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The processing of polar quantifiers, and numerosity perception

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ABSTRACT

We investigated the course of language processing in the context of a verification task that required numerical estimation and comparison. Participants listened to sentences with complex quantifiers that contrasted in Polarity, a logical property (e.g., *more-than-half*, *less-than-half*), and then performed speeded verification on visual scenarios that displayed a proportion between 2 discrete quantities. We varied systematically not only the sentences, but also the visual materials, in order to study their effect on the verification process. Next, we used the same visual scenarios with analogous non-verbal probes that featured arithmetical inequality symbols (<, >). This manipulation enabled us to measure not only Polarity effects, but also, to compare the effect of different probe types (linguistic, non-linguistic) on processing.

Like many previous studies, our results demonstrate that perceptual difficulty affects error rate and reaction time in keeping with Weber's Law. Interestingly, these performance parameters are also affected by the Polarity of the quantifiers used, despite the fact that sentences had the exact same meaning, sentence structure, number of words, syllables, and temporal structure. Moreover, an analogous contrast between the non-linguistic probes (<, >) had no effect on performance. Finally, we observed no interaction between performance parameters governed by Weber's Law and those affected by Polarity. We consider 4 possible accounts of the results (syntactic, semantic, pragmatic, frequency-based), and discuss their relative merit.

1. Introduction

1.1. Numerical tasks and instructions that drive them

This paper describes an attempt to get a glimpse at the manner by which natural language quantifiers are processed in the context of numerical comparison tasks. The study of these processes is important because it might shed light on the nature of the representations that are maintained as such tasks are carried out, and may also provide information about possible interactions between linguistic analysis and numerical comparison.

The rich literature on numerical estimation and comparison in humans typically features paradigms where the task is preceded by a verbal preamble: in many instances, participants are verbally instructed, prior to the beginning of the test session, on how they should perform the task – on how they should respond to each stimulus type. Verbal instructions require linguistic analysis. As numerosity experiments typically focus on non-linguistic processes, they seek to minimize the impact of instructions on processing and performance. As we shall see below, the implicit assumption appears to be that instructions, and representations thereof, are immaterial.

The present study, by contrast, focuses on the impact of verbal instructions on processing, in order to investigate their possible contribution to processing in numerosity tasks. That is, we sought to obtain evidence regarding the interaction (or lack thereof) between on-line linguistic analysis and numerical comparison.

Some details might help to make our goal clear. Numerosity experiments typically feature sequences of quantities. The

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instructions given are often global.[2] Each trial features a sequence, beginning with an image of a fixed *reference* numerosity *r*, which is followed by another image that contains a *comparandum* numerosity *c*, that is varied systematically around *r*. The task requires a comparison between *r* and *c*. For example, Piazza, Izard, Pinel, Le Bihan, and Dehaene (2004) habituated participants to triplets of numerosities of a particular value of *r*; they then presented a fourth numerosity *c*, which varied from one trial to the next. Instructions, given prior to testing, also varied: in one condition, they asked participants to indicate "*whether the fourth set was larger or smaller than the preceding ones*" (Piazza et al., 2004, p. 548).[3] Discrimination depended on both the size of the quantities perceived, and the distance between them. Performance graphs in all conditions were "asymmetrical and better fitted by the integral of a Gaussian on a log scale than on a linear scale" (Piazza et al., 2004, p. 548), leading Piazza et al. to conclude that our internal number line, against which quantity estimations are made, is compressed logarithmically (as predicted by Weber-Fechner's Law, Dehaene, 1997; Dehaene & Changeux, 1993; Nieder & Miller, 2003), where *r*, *c*, are internally represented as means of a normal distribution with a variance that is fixed across all choices of *r*, *c*. Importantly, Piazza et al. (2004) report no effect of instructions on performance.

1.2. Instructional-symmetry and breaks thereof

If numerosity judgments are fully described as the comparison of the internal representations of the reference and comparandum sets, one expects our cognitive system to carry out the same calculation process whether the perceiver is instructed to verify statements that require comparison of *r* to *c*, or *c* to *r* (e.g., *compare r to c* vs. *compare c to r*). Call this property *I(nstructional)-symmetry*.

Now, consider the form and content of verbal instructions. As standard tasks require the estimation of quantities and comparison between them, instructions often feature quantifiers – linguistic elements that express quantity. These words and expressions have long been subject of intense study by linguists, philosophers, psychologists and mathematicians (Barwise & Cooper, 1981; Keenan & Westerstahl, 1997; Lewis, 1970; Mostowski, 1957; Oaksford & Chater, 2007). To see how quantifiers relate to numerosity, we consider the role of quantifiers in the evaluation of truth in the following sentences:

(1) a. She wears **at least 3** rings
b. Is **every** man in the room holding a flag?
c. At least half of the women here are wearing a scarf

In (1a), estimation of the minimal number of rings worn in the scenario must precede truth-value judgment. In (1b), a listener returns "*no*" if there is at least one man without a flag in the room, and "*yes*" otherwise.[4] Sentence (1c) is true just in case the proportion of scarf-wearers among the women in the vicinity of the speaker is half or more. The use of quantifiers is thus intimately related to perceived (sometimes reported or even imagined) numerosity. Experiments with quantifiers indeed involve quantities, and tap both linguistic and numerosity processes (Hackl, 2009; Heim et al., 2012; McMillan, Clark, Moore, Devita, & Grossman, 2005; Moxey & Sanford, 1986; Pietroski, Lidz, Hunter, & Halberda, 2009).

Next, we note that the sentences in (2a–b), that contain *contrary* quantifiers, have the same meaning when the scenario contains circles of 2 colors and nothing else – they are made true and false by the same scenarios of the *r*/*c* variety:

(2)	***Sentence***	***Scenarios***	
		A	B
		2 red circles; 14 black circles	2 black circles; 14 red circles
	a. **More-than-half** of the circles are red	*False*	*True*
	b. **Less-than-half** of the circles are black	*False*	*True*

Indeed, this equivalence has led many studies to treat verbal instructions as a necessary, yet impertinent, component of numerosity experiments, one that merely needs to be properly balanced. For example, Barth, Kanwisher, and Spelke (2003) balanced the comparative quantifiers *more* ... *than* with *fewer* ... *than* in the sentences that they used in a task that required verification against scenarios (Experiment 3). No subsequent analysis attempted to separate performance by the *more*/*fewer* manipulation, presumably because like Piazza et al. (2004)Barth et al. (2003) assumed I-symmetry, namely that equal numbers of sentence tokens of each type renders this contrast orthogonal to the goals of their numerosity test.

However, the quantifiers in (2) *do* contrast in Polarity, a logical property: *More*- and *less-than-half of the circles* license inferences in opposite directions (*many* and *few of the circles*, as well as the comparative quantifiers *more* ... *than* and *fewer*...*than*, are likewise opposed, as illustrated):

(3) Inferences licensed by Monotone Increasing (*a k a* positive) quantifiers
a. **more-than-half** of the students ran fast ⇒ **more-than-half** of the students ran
b. **many** of the students ran fast ⇒ **many** of the students ran
c. there are **more** small circles **than** squares ⇒ there are **more** circles **than** squares

(4) Inferences licensed by Monotone Decreasing (*a k a* negative) quantifiers
a. **less-than-half** of the students ran ⇒ **less-than-half** of students ran fast
b. **few** of the students ran ⇒ **few** of the students ran fast
c. there are **fewer** circles **than** squares ⇒ there are **fewer** small circles **than** squares[5]

The set of students who ran fast is a subset of the set of students who ran. The quantifiers in (3a,b,c) license inferences from the former to the latter are therefore *Monotone Increasing* (or upward entailing), positive quantifiers henceforth. Their *Monotone Decreasing* (or downward entailing) negative counterparts (4a,b,c) license the reverse

[2] Global are given once at the beginning of the experiment; local ones are provided on each trial. Though these different manipulations may have different performance consequences, participants must activate the instructions on every trial, or else they would not know what task they are performing. As we compare between different instruction types within the same mode of presentation, we are legitimized in suppressing the difference between global and local instructions.

[3] Piazza et al. put little emphasis on instructions. They are not entirely clear on whether they gave declarative sentences that called for a True/False response (*the fourth set is smaller*), yes/no questions (*is the fourth set smaller?*), or embedded disjunctive questions (*indicate whether the fourth set is larger or smaller*) that called for a Smaller/Larger response. These differences may have consequences to verification. Yet no cross-instructional difference is reported.

[4] It has been argued that if no man is in the room, the sentence is also true. This position, however, has been contested. In the foregoing, we steer clear from such issues.

[5] Comparatives introduce further complications, but nonetheless feature the Polarity contrast (see below. Cf., also (cf. Schwarzschild, 2008 for a recent review)).