Cognition 137 (2015) 115–136

Contents lists available at ScienceDirect

Cognition

journal homepage: www.elsevier.com/locate/COGNIT

Modeling individual differences in response time and accuracy in numeracy

Roger Ratcliff [a,*], Clarissa A. Thompson [b], Gail McKoon [a]

[a] The Ohio State University, United States
[b] Kent State University, United States

ARTICLE INFO

Article history:
Received 20 December 2013
Revised 1 May 2014
Accepted 17 December 2014

Keywords:
Diffusion model
Response time and accuracy
Numeracy
Number ability
Individual differences

ABSTRACT

In the study of numeracy, some hypotheses have been based on response time (RT) as a dependent variable and some on accuracy, and considerable controversy has arisen about the presence or absence of correlations between RT and accuracy, between RT or accuracy and individual differences like IQ and math ability, and between various numeracy tasks. In this article, we show that an integration of the two dependent variables is required, which we accomplish with a theory-based model of decision making. We report data from four tasks: numerosity discrimination, number discrimination, memory for two-digit numbers, and memory for three-digit numbers. Accuracy correlated across tasks, as did RTs. However, the negative correlations that might be expected between RT and accuracy were not obtained; if a subject was accurate, it did not mean that they were fast (and vice versa). When the diffusion decision-making model was applied to the data (Ratcliff, 1978), we found significant correlations across the tasks between the quality of the numeracy information (drift rate) driving the decision process and between the speed/accuracy criterion settings, suggesting that similar numeracy skills and similar speed–accuracy settings are involved in the four tasks. In the model, accuracy is related to drift rate and RT is related to speed–accuracy criteria, but drift rate and criteria are not related to each other across subjects. This provides a theoretical basis for understanding why negative correlations were not obtained between accuracy and RT. We also manipulated criteria by instructing subjects to maximize either speed or accuracy, but still found correlations between the criteria settings between and within tasks, suggesting that the settings may represent an individual trait that can be modulated but not equated across subjects. Our results demonstrate that a decision-making model may provide a way to reconcile inconsistent and sometimes contradictory results in numeracy research.

1. Introduction

In decision-making tasks, several variables can be used to measure performance. In this article, we use a theory-based approach to investigate how dependent variables interact in decision tasks in the domain of numeracy research. We explain, first, how different dependent variables arise from the same underlying cognitive processes; second, why the value of a dependent measure may or may not be correlated between tasks; and third, why the value of one dependent variable may or may not be correlated with the value of another dependent variable. We show that an understanding of these issues is essential to the evaluation of data in numeracy research and the

* Corresponding author at: Department of Psychology, The Ohio State University, Columbus, OH 43210, United States. Tel.: +1 614 292 7916; fax: +1 614 688 3984.

E-mail address: ratcliff.22@osu.edu (R. Ratcliff).

http://dx.doi.org/10.1016/j.cognition.2014.12.004
0010-0277/

development of theories about numeracy. It is essential for answering questions such as how does the human mind represent numerical information? is there a common representation that is activated and used for all cognitive processes that make use of number? what are these cognitive processes? and what are the representations and processes that underlay children's abilities to learn arithmetic? It also may be essential for elucidating controversies in the numeracy literature. While we ourselves do not resolve any of these controversies, we do show why a decision-making model is required.

When studies have examined correlations between tasks for some dependent measure that is thought to reflect numeracy processes, the results have been mixed. Sometimes correlations are found between symbolic tasks ("is 5 greater than 2") and nonsymbolic tasks ("is the number of dots in one array greater than in another array"), and sometimes not (e.g., De Smedt, Verschaffel, & Ghesquiere, 2009; Gilmore, Attridge, & Inglis, 2011; Holloway & Ansari, 2009; Maloney, Risko, Preston, Ansari, & Fugelsang, 2010; Price, Palmer, Battista, & Ansari, 2012; Sasanguie, Defever, Van den Bussche, & Reynvoet, 2011). Sometimes correlations are found between non-symbolic number tasks and math ability, and sometimes not (e.g., Gilmore, McCarthy, & Spelke, 2010; Gilmore et al., 2010; Halberda, Mazzocco, & Feigenson, 2008; Holloway & Ansari, 2009; Inglis, Attridge, Batchelor, & Gilmore, 2011; Libertus, Feigenson, & Halberda, 2011; Lyons & Beilock, 2011; Mazzocco, Feigenson, & Halberda, 2011; De Smedt et al., 2009; Durand, Hulme, Larkin, & Snowling, 2005; Mundy & Gilmore, 2009; Price et al., 2012).

The inconsistent use of dependent variables compounds these problems. Sometimes accuracy is used, sometimes mean response time (RT), and sometimes the slope of accuracy or RT as a function of the difficulty of a test item. When these variables are not correlated, they can give completely different pictures of number abilities. For example, Gilmore et al. (2011) found little correlation between all combinations of accuracy and RT across a range of symbolic and nonsymbolic tasks. A recent meta-analysis by Chen and Li (2014) reinforces the extent of the problem. For 36 recent studies, they found 21 that used overall accuracy, 9 that used mean RT, 17 that used the Weber fraction (an accuracy-based measure), and 8 that used a numerical distance effect based on RT.

Halberda, Ly, Wilmer, Naiman, & Germine (2012, p. 11116) looked at correlations between two measures, as opposed to the same measure across tasks. One was the Weber fraction (w) and the other was RT. They state that "the Weber fraction and RT are largely uncorrelated ... suggesting they may index independent abilities." Price et al. (2012, p. 54) concurred, saying that "the relationship between RT slope and w is not very strong, which might be explained by the fact that one is a measure of RT while the other is a measure of accuracy."

One of the main arguments we want to make is that accuracy and RT must be explained by the same mechanism, not independent mechanisms. Fig. 1 shows why this is so. The data come from our first experiment: subjects were asked to decide whether the number of asterisks in a display was greater than 50 ("large") or equal or less than 50 ("small"). The top panel shows the probability of responding "large." Responses are highly accurate for the lowest numbers of asterisks and the highest numbers, but not with numbers in between (e.g., 40 asterisks).

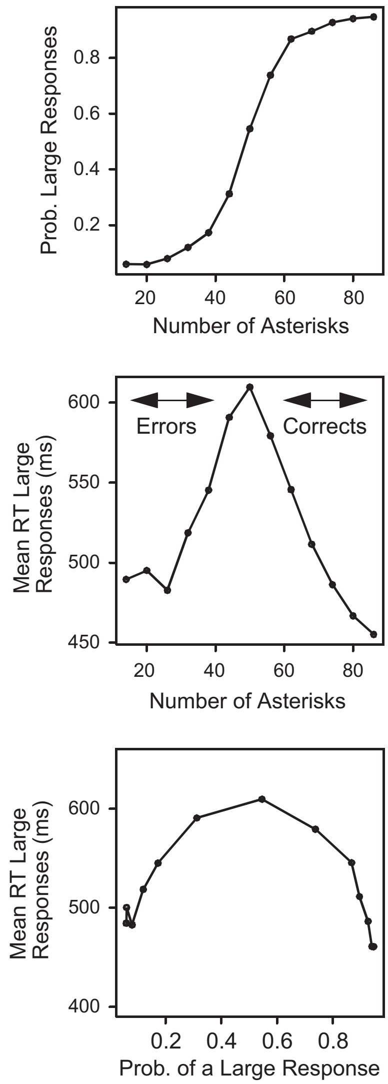

Fig. 1. Plots of probability of a large response against number of asterisks (top panel), mean RT for "large" responses against number of asterisks (middle panel), and mean RT for a "large" response against the probability of a "large" response (bottom panel) in the numeracy discrimination task.

The middle panel shows mean RTs for "large" responses. When they are easy, RTs are short; when they are more difficult, RTs are longer. The right half of the plot shows correct responses and the left error responses (i.e., "large" responses to "small" stimuli). The RTs for correct and error responses mirror each other. The mean RTs for "small" responses show this same pattern.

The bottom panel shows the result that demands an explanation: when mean RTs are plotted against the probability of a "large" response, the data sweep out a single function. When the probability of a "large" response is