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Twist within a somersault

Joanne Mikl*, David C. Rye

Australian Centre for Field Robotics, The University of Sydney, 2006 NSW, Australia

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ABSTRACT

The twisting somersault is a key skill in diving and gymnastics. The components of twist and somersault are defined with respect to anatomical axes, and combinations of multiples of half rotations of twist and somersault define specific twisting somersault skills. To achieve a twisting somersault skill twist must be continuous; otherwise oscillations in twist while somersaulting may be observed. The posture-dependent inertial properties of the athlete and the initial conditions determine if continuous or oscillating twist is observed. The paper derives equations for the amount of somersault required per half twist, or per twist oscillation, without making assumptions about the relative magnitudes of the moments of inertia. From these equations the skills achievable may be determined. The error associated with the common assumption that the medial and transverse principal moments of inertia are equal is explored. It is concluded that the error grows as the number of twists per somersault decreases, when the medial and transverse moments of inertia diverge, and when the longitudinal moment of inertia approaches either the medial or transverse moment of inertia. Inertial property data for an example athlete are used to illustrate the various rotational states that can occur.

1. Introduction

The twisting somersault is an important skill in gymnastics and diving, involving three-dimensional rotation while the athlete is airborne. The skills that may be performed in competition are defined by the Fédération Internationale de Gymnastique in the Code of Points (CoP MAG, 2013; CoP TRA, 2013; CoP WAG, 2013) and by the Fédération Internationale de Natation (FINA) in the Diving Rules (DR FINA, 2015). All skills are defined in relation to the athlete's anatomy and the sporting environment; a somersault is a rotation about a horizontal axis perpendicular to the direction of travel, and twist is a rotation about the longitudinal (head-to-toe) axis of the athlete's body. To be awarded the difficulty points associated with a twisting somersault skill, a diver must simultaneously complete a multiple of a half twist and a multiple of a half somersault, while a gymnast must complete a multiple of a half twist and a multiple of a full somersault. Further, unintended twist in a pure somersault will result in a deduction from the athlete's score. Thus to specify a skill, or to determine if there will be deductions related to unintentional twist, it is necessary to know the amount of twist completed within a somersault.

Previous authors (Batterman, 1968; Frohlich, 1979; Rackham, 1970; Yeadon, 1993) have used rigid body mechanics to describe the phase of the twisting somersault where the majority of the twist occurs. Yeadon states "Once divers and gymnasts have started to twist in a somersault, they often appear to maintain a fixed body configuration. A rigid body may be expected to give a reasonable representation of such phases." (Yeadon, 1993). Better performances are expected

* Corresponding author at: ACFR, The Rose Street Building J04, University of Sydney, 2006 NSW, Australia.
E-mail address: j.mikl@acfr.usyd.edu.au (J. Mikl).

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to show distinct phases, since this allows the concept of "peaking" (George, 2010) to be applied and avoids deductions for "insufficient exactness" (CoP WAG, 2013); it is therefore reasonable to expect athletes to seek to display a quasi-rigid phase where the majority of the twist rotation is completed. Since this phase is entirely airborne there can be no change to the athlete's angular momentum and their centre of gravity will follow a parabolic trajectory. This paper focuses on the quasi-rigid aerial phase, mathematically describing the motion resulting from the constant angular momentum. The rotational aspect of the motion is addressed since gymnastic and diving skills are defined in terms of the rotation achieved rather than the translation that occurred.

Equations giving twist with respect to time (Synge & Griffith, 1959; Yeadon, 1993) and an average somersault rate during a continuously-twisting somersault (Yeadon, 1993) are available in the literature. These equations are written for the special case when the athlete's medial, transverse and longitudinal principal anatomical axes—or their equivalents in a general rigid body—correspond to the axes of the maximum, intermediate, and minimum principal moments of inertia. Gymnastic and diving skills are defined in terms of the twist completed with respect to the somersault rotation, rather than the twist completed in a period of time; consequently, the time-parameterised equations given by Synge & Griffith and by Yeadon do not directly describe specific gymnastic or diving skills.

In general, the principal moments of inertia of the human body will not be equal, although two moments of inertia may be similar in some postures. Furthermore, since the body can adopt many postures it cannot be assumed that the principal axis closest to each anatomical axis is always the maximum, always the minimum or always the intermediate moment of inertia. This paper removes previous assumptions of equality of the medial and transverse moments of inertia (Batterman, 1968; Frohlich, 1979; Rackham, 1970) and assumptions regarding the order of the magnitudes of the principal moments of inertia (Yeadon, 1993).

It is known (Yeadon, 1993) that when the moment of inertia about the principal axis closest to the medial anatomical axis is the intermediate principal moment of inertia then either continuous twist or twist that oscillates about the zero-twist position may be observed. Which occurs depends on the initial orientation of the athlete's longitudinal axis with respect to the angular momentum vector. The case when the transverse moment of inertia is the intermediate-valued principal moment of inertia can be described (Yeadon, 1993) by adding a quarter-twist to the description of the twist when the medial principal moment of inertia is the intermediate moment of inertia. The situation when the moment of inertia about the principal axis closest to the longitudinal anatomical axis is the intermediate moment of inertia has not been described in the literature. The effect of a small amount of initial twist has not been previously discussed. Initial twist could be intentional—the athlete 'cheats'—or caused by unintentional movement preceding the take-off. Good performances will display only small initial twist. This paper describes twelve cases that represent distinct rotational states governed by three factors: which moment of inertia is the intermediate-valued; the initial angle between the longitudinal axis and the angular momentum vector; and the initial twist angle.

The remainder of the paper is organised as follows. First, the equations of motion are derived using rigid body mechanics. The different rotational states possible are then identified, and the equations of motion solved to determine the number of somersaults required per half twist rotation, or per one oscillation of twist. Since an athlete can remove twist (Batterman, 1968; Frohlich, 1979; Rackham, 1970; Yeadon, 1993) the skills that are achievable are those for which the number of half somersaults to be completed is less than the number of somersaults required per half twist multiplied by the number of half twists to be completed. Since assuming that the transverse and medial moments of inertia are equal greatly simplifies the equation for the number of somersaults required per half twist, when such an assumption is reasonable is discussed. One inertial property data set obtained from the literature (Huston, 2009) is used to illustrate the rotational behaviours and aid discussion.

2. Equations of motion

2.1. Frames of reference and orientation angles

Two right-handed frames of reference are defined: the global frame G: $\{t; x, y, z\}$ and the body frame P: $\{t; x, y, z\}$. A prepended superscript is used to denote the frame of reference for a vector. For example, the angular momentum vector measured in P is ${}^{P}H$. The axes of the frames P or G are also distinguished by the prepended superscript. For example, the x-axis of frame P is denoted by ${}^{P}x$ and the y-axis of frame G is ${}^{G}y$.

The global frame G has its origin at the athlete's centre of gravity. The frame G translates with the athlete but does not rotate. Since the origin of G is the centre of gravity and it does not rotate it may be treated mathematically as an inertial frame (Smith & Kane, 1967; Synge & Griffith, 1959). The ${}^{G}z$-axis is vertical. The ${}^{G}y$-axis is the horizontal axis about which a somersault occurs, regardless of whether a pure or a twisting somersault is performed. The ${}^{G}y$-axis will be to the athlete's right in a backward somersault and to the athlete's left in a forward somersault. The ${}^{G}x$-axis is in the direction of travel for both forward and backward somersaults.

The body frame P rotates with the athlete's body. The origin of P is always located at the athlete's centre of gravity, irrespective of the posture that the athlete adopts: its origin is coincident with the origin of G. In the quasi-rigid phase, since the athlete holds a single posture the anatomical landmarks will be fixed with respect to each other and the origin of P. The axes of P are parallel to the principal axes of the body as a whole and named by following anatomy. When the athlete is standing