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A Riemannian geometry theory of human movement: The geodesic synergy hypothesis

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ABSTRACT

Mass-inertia loads on muscles change with posture and with changing mechanical interactions between the body and the environment. The nervous system must anticipate changing mass-inertia loads, especially during fast multi-joint coordinated movements. Riemannian geometry provides a mathematical framework for movement planning that takes these inertial interactions into account. To demonstrate this we introduce the controlled (vs. biomechanical) degrees of freedom of the body as the coordinate system for a configuration space with movements represented as trajectories. This space is not Euclidean. It is endowed at each point with a metric equal to the mass-inertia matrix of the body in that configuration. This warps the space to become Riemannian with curvature at each point determined by the differentials of the mass-inertia at that point. This curvature takes nonlinear mass-inertia interactions into account with lengths, velocities, accelerations and directions of movement trajectories all differing from those in Euclidean space. For newcomers to Riemannian geometry we develop the intuitive groundwork for a Riemannian field theory of human movement encompassing the entire body moving in gravity and in mechanical interaction with the environment. In particular we present a geodesic synergy hypothesis concerning planning of multi-joint coordinated movements to achieve goals with minimal muscular effort.

1. Introduction

Many studies of human movement have focused on kinematic properties but fewer studies have dealt with dynamics due to the lack of mathematical tools for analyzing the complex nonlinear dynamical interactions between the biomechanical degrees of freedom (DOFs) of the human body (Sekimoto, Arimoto, Kawamura, & Bae, 2008). Gravitational and mass-inertia loads on muscles change with posture and with mechanical interactions between the body and the environment. These changes greatly complicate the dynamical relationships between forces generated by muscles and resulting body movements. Even rotation about a single joint requires a coordinated activation of many muscles throughout the body to compensate for these dynamical interactions and to prevent movement about other DOFs. Inertial, centrifugal, and coriolis reaction forces have to be anticipated by the nervous system, especially during fast multi-joint movements.

Further complicating the picture is the fact that most functional multi-joint movements require the changing joint-angles to be appropriately coordinated to achieve performance goal(s). To produce this coordination the nervous system has to

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couple changing joint-angles together by introducing nonlinear dynamical constraining relationships between them. We refer to such a set of constraining relationships as a *movement synergy*. Because there are many more joint-angles than performance variables, many different combinations of joint-angles and different coordination of those joint-angles can be employed to achieve the same performance goal(s).

How does the nervous system determine which particular coordination of joint-angles to use? One possibility is that it selects the coordination (i.e., movement synergy) that achieves performance goal(s) with minimum demand for muscular effort. It makes sense from an evolutionary point of view that the nervous system might have evolved strategies to achieve goals, such as catching prey and escaping predators, with minimum demand for muscular effort (O'Dwyer & Neilson, 2000). Moreover, everyday movements appear to be constrained by the imperative to optimize metabolic economy (see Sparrow & Newell, 1998 for review). Any computational theory of motor control that seeks to explain how such a movement synergy might be implemented must take into account the multi-linked nonlinear dynamics of the human body in interaction with the environment. As indicated earlier, this raises the question of what mathematical tools and computational techniques are required to handle such nonlinear dynamics.

We contend that Riemannian geometry provides the most appropriate mathematical framework for developing computational models of multi-joint human movement. Riemannian geometry represents motion in a nonlinear (curved) space known as a Riemannian manifold. The geometry of this curved space differs considerably from the intuitively well-understood geometry of linear Euclidean space. For example, the notion that parallel lines never meet is a Euclidean idea that does not hold in curved Riemannian space. Concepts concerning distances, areas, volumes, straight lines, angles between lines, velocities, and accelerations derived from Euclidean geometry all have to be modified. As has been shown already (Biess, Flash, & Liebermann, 2011; Biess, Liebermann, & Flash, 2007), some of the paradoxes and contradictions in existing computational theories of human movement (Hermens & Gielen, 2004) based on linear Euclidean notions can be resolved when reformulated in Riemannian space. Furthermore, as pointed out by Biess (2013), an investigator's choice of coordinates can be problematic in models of human movement. No consistent inferences can be drawn because concepts such as anisotropy and orthogonality of covariance matrices, for example, are coordinate dependent. A Riemannian geometry framework allows theories of human movement to be expressed in a coordinate independent manner because tensor equations are invariant under coordinate transformations; that is, if a tensor equation equals zero in one coordinate system then it will equal zero in all coordinate systems.

Once a Riemannian framework for human movement is established, the theorems and propositions of that geometry offer the possibility of obtaining new insights into motor behaviour. In this paper we lay the groundwork for a Riemannian model of the entire human body moving in a gravitational field in mechanical interaction with the environment. In particular, we will use Riemannian geometry to show theoretically how the nervous system can coordinate movements to achieve goals with minimum demand by muscles for metabolic energy.[1] We call this the *geodesic synergy hypothesis*.

2. Background

2.1. The human body as a multi-linked mechanical system

From a biomechanical point of view the human body is a multi-linked mechanical system with multiple biomechanical DOFs. Sekimoto et al. (2008) demonstrated that the inertia-induced movement of a multi-linked mechanical system is characterized by geodesic curves in a Riemannian manifold. Similarly, Bullo and Lewis (2005) showed that motion of multi-linked mechanical systems can be treated using Riemannian geometry and that motion in the absence of an external force corresponds to geodesic trajectories in the Riemannian manifold. Arimoto (2010) showed that the space spanned by the DOFs of multi-joint robots can be regarded as a Riemannian manifold with a Riemannian metric equal to the robot's mass-inertia matrix. As will be developed in Section 3, the curvature of a Riemannian manifold for a multi-linked mechanical system is attributable to the changing Riemannian metric (i.e., changing mass-inertia matrix).

2.2. Previous applications of Riemannian geometry in studies of human movement

Handzel and Flash (1999) were early to present the case for using geometric methods in the study of human motor control. They pointed out that the spaces of motor DOFs had not so far been dealt with in a satisfactory way. All too often, they argued, motor DOFs were treated as a collection of *a priori* unrelated variables represented as a linear Euclidean space with the nonlinear geometric structure inherent in multi-linked systems ignored. Consequently, measurements and computations of lengths of paths were distorted leading to erroneous conclusions. These limitations are overcome, they explained, by employing spaces that do not have a linear structure. In general, these nonlinear spaces are represented by curved differentiable (smooth) manifolds, the basic objects of Riemannian geometry.

[1] We consider minimizing muscular effort and minimizing demand by muscles for metabolic energy to be the same as minimizing the net tensions developed by muscles. This is distinct from *sense* of muscular effort (an aspect of kinesthesia) which correlates with the amount of neural drive to the muscles (Gandevia, 1987; McCloskey, 1981; O'Dwyer & Neilson, 2000).