Journal of Behavioral and Experimental Finance 11 (2016) 13–17

Contents lists available at ScienceDirect

Journal of Behavioral and Experimental Finance

journal homepage: www.elsevier.com/locate/jbef

Full length article

Prospect theory and portfolio selection

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ARTICLE INFO

Article history:
Received 16 December 2015
Received in revised form
15 April 2016
Accepted 15 May 2016
Available online 6 June 2016

JEL classification:
G11

Keywords:
Portfolio choice
Prospect theory
Kinked linear utility

ABSTRACT

We examine prospect theory portfolios in asset allocation settings that include riskfree lending and borrowing, subject to margin constraints, and short sales restrictions on risky assets. In static settings, we focus on myopic loss aversion, which assumes loss averse investors are willing to take more risk if they evaluate their investment performance infrequently. The results show the portfolios, including those of the investor with a loss aversion coefficient of 2.25, are extremely unstable across decision horizons. In dynamic settings, the portfolios of investors with loss aversion on the order of two perform well. But in some instances the house money effect, where the position of the kink and the investor's loss aversion changes with gains and losses, has a large negative impact on the wealth of these investors.

1. Introduction

Based on experimental evidence, which indicated individuals do not obey the expected utility axioms, Kahneman and Tversky (1979) developed prospect theory. Utility is defined over gains and losses. The utility (or value) function is characterized by a kink (a point of non-differentiability) usually at zero gains or losses. The key feature is loss aversion, where individuals exhibit a greater sensitivity to losses than to gains. Tversky and Kahneman (1992) extended the theory and estimated that the value function is slightly concave (convex) over gains (losses) with a loss aversion coefficient of 2.25.

Benartzi and Thaler (1995) helped bridge the gap between experimental and real world investment settings by calculating portfolios based on prospect theory utility functions. They added richness to the model assuming loss averse investors are willing to take more risk if they evaluate their investment performance infrequently—a characteristic known as myopic loss aversion. In a static setting, they calculated that an investor with a loss aversion parameter of 2.25 who evaluates his portfolio annually holds a reasonable 50:50 bond stock mix.

In a dynamic setting, Barberis et al. (2001) added two important ideas. First, gains and losses are measured relative to the riskfree rate of return rather than zero. Second, following Thaler and Johnson (1990), loss aversion and the kink point change with prior gains and losses. The idea that risk aversion goes down after prior gains is termed the house money effect, reflecting gamblers' increased willingness to bet when ahead.

Based on that evidence, Barberis and Thaler (2003) suggested that of all the non-expected utility theories, prospect theory may be the most promising for financial applications.

However, there may be an extreme-weights problem for prospect theory investors – as well as mean–variance investors – in highly unrealistic opportunity sets where there is either unlimited borrowing at a riskless rate and/or unlimited short sales opportunities. See He and Zhou (2011) and De Giorgi et al. (2010) for example.

The more relevant question addressed in this paper is: How do prospect theory portfolios perform in realistic investment settings?

In static asset allocation settings, we focus on myopic risk aversion, examining a wide range of loss averse investor types, paying special attention to the investor with a loss aversion coefficient of 2.25. In these realistic opportunity sets that include riskfree lending and borrowing, subject to margin constraints, prospect theory portfolios are extremely unstable.

In dynamic settings, we examine the out-of-sample characteristics of prospect theory portfolios. We pay special attention to whether the results change when gains and losses are measured relative to the riskfree rate of return rather than zero and when loss aversion and the kink point change with prior gains and losses.

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http://dx.doi.org/10.1016/j.jbef.2016.05.002

The paper proceeds as follows. Section 2 outlines how we formulate and solve the prospect theory investment problem, and how the model is employed in real world asset allocation settings. Section 3 describes the data. Section 4 reports the empirical results. Section 5 contains a summary and concluding comments.

2. The model

Like (Benartzi and Thaler, 1995) and (Barberis et al., 2001), among others, we formulate the prospect theory investment problem in terms of kinked linear utility. We do so because the S-shaped prospect theory utility function is non-concave, which may lead to multiple and differing local optima. The prospect theory investment problem with kinked linear utility is

$$\max_{\mathbf{x}_t} \begin{pmatrix} \sum_s \pi_{ts}(1 + r_{pts}(\mathbf{x}_t)) & \text{if} \quad (1 + r_{pts}(\mathbf{x}_t)) > \hat{w} \\ \sum_s \pi_{ts} l(1 + r_{pts}(\mathbf{x})) & \text{if} \quad (1 + r_{pts}(\mathbf{x}_t)) \leq \hat{w} \end{pmatrix}, \tag{1}$$

subject to

$$x_{it} \geq 0, \quad \text{all } i, \qquad x_{Lt} \geq 0, \qquad x_{Bt} \leq 0, \tag{2}$$

$$\sum_i x_{it} + x_{Lt} + x_{Bt} = 1, \tag{3}$$

$$\sum_i m_{it} x_{it} \leq 1, \tag{4}$$

where $r_{pts}(\mathbf{x}_t) = \sum_i x_{it} r_{its} + x_{Lt} r_{Lt} + x_{Bt} r_{Bt}^d$ is the *ex ante* return on the portfolio in period t if state s occurs, $l \geq 1$ is the slope associated with losses, $\hat{w}$ is the kink point (set equal to one in the simplest case with gains and losses measured relative to a zero rate of return). The fractions of wealth invested in risky asset i, lending, and borrowing in period t are given by x_{it}, x_{Lt}, and x_{Bt}, respectively, where $\mathbf{x}_t = (x_{1t}, \ldots, x_{nt}, x_{Lt}, x_{Bt})'$. In addition, r_{it} and r_{Lt} are the rates of return on asset i and the riskfree asset and r_{Bt}^d is the interest rate on borrowing at the time of the decision at the beginning of period t. Finally, m_{it} is the initial margin requirement for asset category i in period t expressed as a fraction, where $0 \leq m_{it} \leq 1$, and π_{ts} is the probability of state s at the end of period t, in which case the random return r_{it} will assume the value r_{its}. Constraint (2) rules out short sales and ensures that lending (borrowing) is a non-negative (non-positive) fraction of capital. Constraints (3) and (4) are the budget and margin constraints, respectively.

We solve the system given by the objective function (1) subject to the constraints (2)–(4) using the algorithm described in Best et al. (2014) and Best and Zhang (2011). In order to overcome the problem of non-differentiability at the kink point Best, Grauer, Hlouskova and Zhang transformed the kinked linear utility problem into a higher dimensional linear program which is differentiable. Then they provided an efficient algorithm that solves the problem in a smaller dimensional space.

Static and dynamic problems are examined. In both cases historical data are employed to estimate the return distribution. It is assumed that a joint realization of the returns on the risky assets at a point in time is an equally probable state-of-nature. The static investment problems are solved once using the complete set of monthly, quarterly or annual return data.

The dynamic problems use quarterly data. We follow the base case scenario employed in previous power utility portfolio selection studies. See, for example, Grauer and Hakansson (1987). At the beginning of quarter t, the kinked linear utility portfolio problem for that quarter uses the following inputs: the (observable) riskfree return for quarter t, the (observable) call money rate plus one percent at the beginning of quarter t, and the (observable) realized returns for the risky asset categories for the previous 32 quarters. Each joint realization in quarters $t - 32$ through $t - 1$ is assigned a probability 1/32 of occurring in quarter t. Thus, estimates are obtained on a moving basis and used in raw form without adjustment of any kind.

With these inputs in place, the portfolio weights for the various asset categories and the proportion of assets either borrowed or loaned are calculated by solving the system given by the objective function (1) subject to the constraints (2)–(4). At the end of quarter t, the realized returns on the risky assets are observed, along with the realized borrowing rate r_{Bt}^r. Then, using the weights selected at the beginning of the quarter, the realized return on the portfolio chosen for quarter t is recorded. The cycle is then repeated in all subsequent quarters. All reported returns are gross of transaction costs and taxes and assume that the investor in question had no influence on prices.

The dynamic behavior of the kink point is modeled in three ways. In the simplest case, the kink point is set equal to one. Gains and losses measured relative to a zero rate of return and the loss slope is constant over time. In a second case, the kink point is set equal to one plus the (changing) riskfree lending rate. Gains and losses are measured relative to the riskfree lending rate and the loss slope is constant over time. Finally, the house money effect, where the investor's loss aversion and kink point are assumed to change with gains and losses, is modeled in a number of ways. The analysis begins with the kink point set equal to one and a given loss slope. If there is a gain greater than 10% (5%) at the end of the quarter, the kink point shifts to 0.9 (0.95) with the loss slope remaining at its given value. If there is a loss greater than 5% (2.5%) at the end of the quarter, the kink point returns to one and the loss slope takes on one of a number of values ranging from 1.05 to 2 times the investor's original loss slope.[1]

3. The data

The data used to estimate the probabilities of the next period's returns on risky assets, and to calculate each period's realized returns on risky assets, come from several sources. The returns series for an index of long-term government bonds, common stocks (the S&P 500), and an index of small stocks are from the Ibbotson Associates' Stocks, Bonds, Bills, and Inflation dataset available through Morningstar. The riskfree asset is assumed to be the 30-day, 90-day, or annual U.S. Treasury bills maturing at the end of the month, quarter, or year as the case may be. The *Survey of Current Business* and the *Wall Street Journal* are the sources. The borrowing rate is assumed to be the call money rate plus one percent for decision purposes (but not for rate of return calculations). The applicable beginning of period decision rate, r_{Bt}^d, is viewed as persisting throughout the period and thus as riskfree. For 1934–76, the call money rates are obtained from the *Survey of Current Business*. For later periods, the *Wall Street Journal* and Bloomberg are the sources. Finally, margin requirements for stocks are obtained from the *Federal Reserve Bulletin*.

4. The results

4.1. Descriptive statistics for the asset categories

To save space we focus on an asset allocation universe, which consists of long-term government bonds (GB), the S&P 500 index (CS), a small stock index (SS) and riskfree lending or borrowing, in both the static (in-sample) and dynamic (out-of-sample) settings.

[1] Of course, this just scratches the surface. For example, gains or losses might be calculated over a period longer than a quarter. Or the size of the gains and losses that trigger changes in the kink point and loss slope might depend on the investor's level of loss aversion.