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The numerical method for the coverage interval determination in the conducted emission measurements

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ABSTRACT

This paper presents a numerical method for the coverage interval determination of the output variable while knowing the probability density functions of two non-dependent input variables. The coverage interval is derived from distribution function. In order to obtain the probability density function of the output variable, which is of mixed distribution type, the numerical combined method is applied, consisted of the Monte Carlo method and the modified least-squares method. The proposed method is applied for the symmetric distributions in the conducted emission measurements. The validation of the combined method showed its satisfactory level of accuracy.

1. Introduction

The Guide to the Expression of Uncertainty in Measurement – GUM Supplement 1 [1] sets a current standard for expressing the measurement uncertainty. According to this document, a measure of uncertainty is the coverage interval. The coverage interval is an interval that contains the value of a quantity with a stated probability, on the basis of available information [1,2]. In addition, the coverage interval is connected with the probability distribution of the output variable (measurand). The term 'coverage interval' [1] should not be mixed with the term 'confidence interval' [3]. This latter term is used in statistics. Several approaches can be used to determine a coverage interval for the measurand: (1) principle of maximum entropy, (2) Bayesian treatment and (3) propagation of distributions [4]. The principle of maximum entropy is based on a unique selection of probability density function (PDF) for the measurand from all PDFs that have specified properties (expectation, standard deviation and intervals) on condition that the PDF is non-zero. Bayesian treatment is based on a probabilistic model for the measurement which is expressed as a function of probability. It is used to update prior information about the measurand (a prior PDF, a posterior PDF) [4]. Propagation of distributions is based on a model function which is used to relate the measurand to model input quantities [4]. This method may be applied in few ways: by analytical methods, uncertainty propagation, numerical methods [1]. Many publications treat each of these methods [4–10].

The coverage interval can be derived from the cumulative distribution function (CDF). Namely, the coverage interval is calculated by the propagation of probability distributions through a measurement model [10,11]. The fundamental purpose of the numerical method is to obtain a coverage interval of the model output quantity, which is dependant on two input variables [12]. In the case of a linear dependence between input and output variables, the probability density function (PDF) of output variable (measurand) can be derived by using a Monte Carlo method (MCM) and a modified least-squares method, i.e. combined method [11]. MCM and the modified least-squares method application give a mixed distribution, which is, in fact, a numerical approximation of the measurand PDF. Since the cumulative distribution function represents the integral of its corresponding PDF [1], the combined method produces the numerical approximation of the measurand CDF. According to this, combined, method the measurand probability density function behaves symmetrically. This sort of problems are met in the conducted emission measurements [12,13].

The aim of this paper is to present a numerical method for the coverage interval determination of the output variable, while knowing the probability density functions of two non-dependent input variables.

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2. Evaluation of the cumulative distribution function

CDFs are calculated by the propagation of probability distributions by using the measurement model. The method used in this paper consists of following steps [11]: (1) Evaluation of the measurand PDF; (2) estimation of PDF parameters; (3) determination of the measurand CDF; and (4) the measurand coverage interval determination.

The evaluation of measurand PDF is performed by using a MCM and a modified least-squares method (combined method) [11]. The combined method demands numerical estimation of measurand probability density function (point estimates parameters of the mixed distribution) [11,12]. The procedure for obtaining these parameters is described in detail [11].

The measurand cumulative distribution function estimation is based on the set of values [11]. Therefore, the numerical method is applied in three different cases including two non-dependent input variables. The first case considers two input variables which follow normal distributions. The second case considers two input variables of which one follows normal distribution and the other follows rectangular distribution. The third case considers two input variables of which one follows normal distribution and the other follows triangular distribution [11].

The simulation was performed in the Visual Basic 6.0. The data for simulations were as follows: (a) the events number $N = 10^6$, (b) the data number $n = 5000$, (c) risk conformity $\alpha = 0.025$, and (d) the mixed coefficient $\varepsilon = 0.5$. In addition, the theoretical curve (GUM method), estimated curve (combined method) and empirical curve were compared.

2.1. The mixed normal–normal CDF

CDF of the output variable which is of mixed normal–normal type, $F(x)$, and the corresponding CDFs for the input variables, $F_1(x)$ and $F_2(x)$, are given by the Eqs. (1)–(3), respectively [11]:

$$F(x) = \varepsilon F_1(x) + (1-\varepsilon)F_2(x), \quad 0 \leqslant \varepsilon \leqslant 1 \tag{1}$$

$$F_1(x) = \frac{1}{s_1\sqrt{2\pi}}\int_{-\infty}^{x} \exp\left[-\frac{1}{2}\left(\frac{x-m_1}{s_1}\right)^2\right]\mathrm{d}x, \quad -\infty < x < +\infty,\ s_1 > 0 \tag{2}$$

$$F_2(x) = \frac{1}{s_2\sqrt{2\pi}}\int_{-\infty}^{x} \exp\left[-\frac{1}{2}\left(\frac{x-m_2}{s_2}\right)^2\right]\mathrm{d}x, \quad -\infty < x < +\infty,\ \ s_2 > 0 \tag{3}$$

where m_1 and m_2 are mean values of the first and the second normal distribution, respectively, s_1 and s_2 are standard deviations of the first and the second normal distribution, respectively, ε – mixed coefficient of these distributions, $\varepsilon \in [0, 1]$.

In Fig. 1, which shows theoretical curve, empirical curve and estimated curve obtained by Eqs. (1)–(3), it can be seen that all three curves are in a good agreement.

2.2. The mixed normal–rectangular CDF

CDF of output variable which is of mixed normal–rectangular type, $F(x)$, is given by the Eq. (1), and the corresponding CDFs of the input variables, $F_1(x)$ and $F_2(x)$, are given by the Eqs. (4) and (5), respectively:

$$F_1(x) = \frac{1}{s\sqrt{2\pi}}\int_{-\infty}^{x} \exp\left[-\frac{1}{2}\left(\frac{x-m}{s}\right)^2\right]\mathrm{d}x, \quad -\infty < x < +\infty, \quad s > 0 \tag{4}$$

$$F_2(x) = \frac{x}{b-a} - \frac{a}{b-a}, \quad a \leqslant x \leqslant b, \quad b > a \tag{5}$$

where m and s are mean value and standard deviation of the normal distribution, while a and b are lower and upper limits of a rectangular distribution.

Fig. 2 shows the simulation results obtained for the mixed normal–rectangular CDF. In this case, it can be seen that the theoretical curve and the empirical curve are in a good agreement, while the estimated curve has some discrepancy from the theoretical curve when approaching limits. This difference is the consequence of the stochastic nature of the Monte Carlo method and the modified least-squares method, i.e. the result of evaluation of parameters of the mixed distribution (whose values are pseudorandom) and the number N of iterations (the events number).

2.3. The mixed normal–triangular CDF

As in the previous case, the mixed normal–triangular CDF, $F(x)$, is given by Eq. (1) while the CDFs of the input variables, $F_1(x)$ and $F_2(x)$, are given by the Eqs. (6) and (7), respectively:

$$F_1(x) = \frac{1}{s\sqrt{2\pi}}\int_{-\infty}^{x} \exp\left[-\frac{1}{2}\left(\frac{x-m}{s}\right)^2\right]\mathrm{d}x, \quad -\infty < x < +\infty, \quad s > 0 \tag{6}$$

$$F_2(x) = \begin{cases} \frac{2(x-a)^2}{(b-a)^2}, & a \leqslant x \leqslant c, \\ 1 - \frac{2(b-x)^2}{(b-a)^2}, & c < x \leqslant b, \end{cases} \quad c = \frac{a+b}{2} \tag{7}$$

where m and s are mean value and standard deviation of the normal distribution, a and b are lower and upper limits of a triangular distribution and c is mode of the triangular distribution.

Fig. 3 shows the simulation result obtained for the mixed normal–triangular CDF. As can be seen, the theoretical curve, estimated curve and the empirical curve are in a good agreement, in the right half of the curve. In the left half of the curve, the estimated curve has some discrepancy from the theoretical curve. This difference is the consequence of the stochastic nature of the Monte Carlo method and the modified least-squares method, as stated in Section 2.2.

3. Determination of the coverage interval

The definition of the coverage interval is given in two forms [1,5]. The first statement refers to the probabilistically symmetric coverage interval or statistical coverage interval: "coverage interval for a quantity such that the probability that the quantity is less than the smallest value in the interval is equal to the probability that the quantity is greater than the largest value in the interval" [1,5,14]. Consequently, determination of the 'probabilistically symmetric coverage interval' assumes that values are symmetrically distributed in respect to their midpoint. The other statement relates to the shortest coverage interval: "coverage interval for a quantity with the shortest length among all coverage intervals for that quantity having the same coverage probability" [1,5,14]. Determination of 'the shortest coverage interval' assumes that the shortest interval, in the set of all coverage intervals with the same probability, has been found.

Propagation of distributions is one of the approaches for determining the interval coverage. In addition, there are several ways (analytical methods, uncertainty propagation, and numerical methods) that are used. The analytical method may be used when model of the measurand is a linear or a linearized. This method depends on an approximation of the convolution of standard distributions (e.g. Student's, normal, rectangular, triangular or trapezoidal) for input variables [5–7]. The uncertainty propagation is based on a construction of a linear approximation to the model function. In addition, a coverage interval based on characterizing