Journal of Behavioral and Experimental Finance 9 (2016) 56–62

Contents lists available at ScienceDirect

Journal of Behavioral and Experimental Finance

journal homepage: www.elsevier.com/locate/jbef

Full length article

Numeraire independence and the measurement of mispricing in experimental asset markets

Owen Powell

Department of Economics, University of Vienna, Austria

ARTICLE INFO

Article history:
Received 22 July 2015
Accepted 19 November 2015
Available online 15 January 2016

JEL classification:
C43
C90
D49
D84
G14

Keywords:
Asset markets
Averaging methods
Mispricing measures

ABSTRACT

Mispricing (the difference between prices and their underlying fundamental values) is an important characteristic of experimental markets. The literature on the topic consists of many different measures. This state of affairs is unsatisfactory, since it is not clear to which extent results are sensitive to the choice of measure. This paper shows that numeraire independence is an important condition not satisfied by previous measures. Furthermore, under additional assumptions it can be shown that the geometric mean is the only such aggregation function to satisfy numeraire independence. This leads to the proposal of two new measures of mispricing, *Geometric Deviation* (for overpricing) and *Geometric Absolute Deviation* (for absolute mispricing). An application illustrates the potential impact of these new measures on previous experimental results.

1. Introduction

Markets are characterized by several metrics, such as their allocative efficiency, liquidity and trading volume. One such particularly well-studied property is *mispricing*: the extent to which prices (as a measure of subjective preferences) deviate from fundamental values. This property, also known as price efficiency and price discovery, has been studied extensively in the context of experimental asset markets (see Palan, 2013 for a review).

One notable feature of the literature on mispricing in experimental asset markets is the large number of measures which have been used. This state of affairs is unsatisfactory, since it is not clear to which extent results are driven by the choice of measure. Ideally, a set of justifiable conditions would be found that define a *unique* measure, or at least reduce the set of viable alternatives.[1] Stöckl et al. (2010, SHK) attempt to address this issue by proposing a set of four such conditions. They show that none of the measures in use at the time simultaneously satisfied all of these four conditions, and use this as motivation to propose new measures. Nevertheless, the conditions themselves do not identify a *unique* measure of mispricing. For example, the SHK measures use an arithmetic mean to aggregate over sets of prices and fundamentals, but no explanation is given for why this procedure, as opposed to any other, is used.

In fact, as the following section shows, the arithmetic mean of relative values (such as prices and fundamentals) suffers from a particular sensitivity: it is not independent

[1] This idea is closely related to the formation of an ideal price index (Fisher, 1922), index numbers (Diewert, 1979) and functional equations (Eichhorn, 1978).

E-mail address: opowell@gmail.com.

http://dx.doi.org/10.1016/j.jbef.2015.11.002

of the choice of numeraire. The choice of numeraire (and hence representation) is arbitrary, and does not affect the implied rate of exchange between two assets (for example, the relative values 2 EUR/\$and 0.5 \$/EUR use different numeraires to represent the same rate of exchange). However, the *arithmetic mean* of such values *is* sensitive to the choice of numeraire, and therefore so are arithmetic mean-based measures such as those proposed by SHK.[2]

This paper extends the work of SHK by using the concept of numeraire independence to identify a *unique* measure of mispricing. First, it shows that replacing the arithmetic mean by its geometric counterpart resolves the issue of numeraire sensitivity. Furthermore, under additional assumptions the geometric mean is the *only* such measure with this property. Finally, the geometric mean has the additional benefit of being invariant to whether normalization is carried out at the individual observation or aggregate level. As a result, new measures are proposed that satisfy numeraire independence.

The rest of the paper is structured as follows. Section 2 reviews the SHK conditions and measures. Section 3 examines the property of numeraire independence and shows that (under certain conditions) the geometric mean is the only aggregation function that satisfies this condition. Section 4 illustrates the application of the theory to experimental asset markets and Section 5 concludes.

2. Background

Consider a market in which two assets are exchanged over a series of N fixed time intervals. Price indices p_i give an estimate of the subjective value of the assets in interval $i \in 1, \ldots, N$. The assets also have an objective fundamental value v_i, which is determined by the relative returns from holding each of the assets indefinitely from time i until the end of the market.

This paper adopts the following definition of "mispricing":

Definition 1 (*Mispricing*)**.** On average over time, how far prices for an asset differ from its fundamental value.

Two variations of this concept are considered, based on how the difference between prices and fundamentals is counted: (1) "absolute mispricing", which only considers the magnitude of the difference, and (2) "overpricing", which considers both the magnitude *and* direction of the difference.[3]

SHK identify a set of four conditions for measures of both types of mispricing, namely that a measure (1) relates FV and prices, (2) be monotone in the difference between FV and prices, (3) be independent of the number of intervals, and (4) be independent of the (absolute) level of the FV (p. 286).They show that no previous measure in the literature simultaneously satisfies these conditions, and as a result propose new measures that have since become *de-facto* standard in the field. Their proposed measure of overpricing is *Relative Deviation* (*RD*):

$$RD(p, v) = \frac{\frac{1}{N}\sum_i p_i - v_i}{\frac{1}{N}\sum_i v_i}. \tag{1}$$

RD captures the overpricing of shares relative to cash, and does so using aggregate measures of prices and fundamentals based on the arithmetic mean. Their proposed measure of absolute mispricing, *Relative Average Deviation* (*RAD*), is similar, but uses the absolute value of price deviations.

Part of the popularity of the SHK measures can be ascribed to the fact that they follow (for the most part) previous convention. The arithmetic mean is used to average across intervals, although no explanation is given for this choice.[4] In fact, the only substantial difference between SHK and some previous measures is driven by their fourth condition, that a measure be independent of the absolute value of the arithmetic mean of the fundamental value. Since no previous measures satisfy this condition, this is used to motivate their own measures, which are essentially normalized versions of other measures.

The last two SHK conditions may be interpreted as independence conditions. They say that mispricing be independent of certain nominal variables, namely (1) the number of intervals, and (2) the absolute average level of the fundamental value. By construction, aggregate measures increase with the number of observations, therefore this needs to be controlled for. By the same token, it should be expected that prices will increase proportionally to fundamentals. If the shares traded in one setting are worth exactly twice as much as those in another (as measured by the fundamental value per share), then *ceterus paribus* it makes sense to also expect the prices in the first case to be twice the prices in the second. Normalizing by the aggregate level of the fundamental value is also consistent with insuring that mispricing is not affected by nominal changes in accounting units (from cents to Euros, for example).

It is less clear whether the fundamental value should be controlled for at the individual observation or aggregate level. SHK propose only normalizing at the aggregate level (p. 290: "We normalize by [aggregate] $|FV|$ to reduce the impact of different FV market designs on the measure".). However, using this same logic, a similar argument can be made for making the adjustment at the *interval* level as well. The logic is exactly the same: prices should be proportionally higher in intervals with higher fundamental values. Price deviations that occur for higher fundamental values should be "discounted", given that they represent less severe mispricing than equivalent deviations that occur during intervals of low fundamental values. Therefore the argument can be made that interval mispricing also be measured proportional to the interval

[2] For this reason, many fields avoid the use of the arithmetic mean: general equilibrium modeling (Flemming et al., 1977), housing prices (Chmelarova and Nath, 2010), exchange rates (Brodsky, 1982; Papell and Theodoridis, 2001), agriculture (Paudel and McIntosh, 2005), psychology (Aczél and Saaty, 1983), and technical performance (Fleming and Wallace, 1986).

[3] SHK refer to the first type as "mispricing", and the second as "overvaluation".

[4] One partial motivation could be a variation of their monotonicity condition: strict monotonicity would rule out the median.