Measurement 91 (2016) 509–518

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Data correlation analysis for optimal sensor placement using a bond energy algorithm

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ARTICLE INFO

Article history:
Received 29 October 2015
Received in revised form 18 May 2016
Accepted 25 May 2016
Available online 26 May 2016

Keywords:
Structural health monitoring
Optimal sensor placement
Data correlation
Bond energy algorithm

ABSTRACT

Optimal sensor placement is one of the crucial and fundamental factors for constructing a cost-effective structural health monitoring system and is related to the effective evaluation of the state of the structure. Structural responses are correlated to some extent, as the structural behavior is continuous. Based on the above two considerations, the question arises of how to obtain the maximum amount of information for understanding the structure using measurements from limited sensors and not be limited to direct monitoring at the placements where the limited sensors are located. Data correlation analysis for optimal sensor placement is proposed using a bond energy algorithm, in which the objectives, such as structural response evaluation covering the maximum structural responses using measurements from sensors located at the optimal placements, are taken into account. The data correlation analysis is conducted for the structural responses, and the correlation matrix is established. Furthermore, the optimal sensor placements and the correlation of the responses at element locations can be determined using the bond energy algorithm. A Schwedler single-layer spherical lattice dome-like structure, which is a common large space steel structure, is used to simulate the structural responses and verify the effectiveness of the proposed method by discussion of different scenarios of parameter selection.

1. Introduction

With the development of structural health monitoring technology, more attention has been given to its application in large-span space structures that have numerous nodes and components. Considering the limitations of economic and site conditions, it is impossible to place sensors at every position [1–3]. Therefore, it has become an important research question to optimize the finite number of sensors to obtain as much information as possible regarding the structure in the field of structural health monitoring. Furthermore, such prophase work is indispensable to implementing a structural health monitoring system [4].

At present, there are many ways to obtain optimal sensor placements. Kammer [5] proposed an effective independence method of optimal sensor placements for large space structures. In this method, the position was sorted based on the contribution of linear independence of the target mode components; by deleting points with little contribution to the independence of the target modes, the limited sensors can be chosen to collect as much information as possible for obtaining modal information. The modal strain energy method was proposed to analyze the structure, in which the larger modal strain energy in terms of degrees of freedom coincides with the location of the larger structural response, and the sensors placed in these positions therefore benefit structural modal identification [6]. The spline function interpolation method is another solution to obtain the optimal sensor placements, in which the structural responses of the other points and limited measure node modal responses are first obtained. Then, the minimum interpolation error is used to determine the optimal sensor placements for a simple beam [7]. To obtain the modal dynamic information used in fingerprint update identification and model update for the structural health monitoring of bridges, Cui and Yuan [8] proposed a methodology of optimal sensor placements with the purpose of selecting key measuring points from a complex degree of freedom structural model. Guyan [9] proposed the model reduction method to determine optimal sensor placements for reflecting the low-order vibration modes of the structure. Master-slave constraint equations for degrees of freedom were substituted into the kinetic energy or strain energy expression of the system to reduce the degrees of freedom. The structural degrees of freedom can be divided into primary and secondary degrees of freedom, and the shrinkage stiffness or mass matrix can be formed while maintaining the primary degrees of freedom.

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http://dx.doi.org/10.1016/j.measurement.2016.05.089
0263-2241/

Lu and Teng [10] proposed a method for optimal sensor placement using a distance measure matrix and synthesized support degree, in which the number and placement of accelerometers were determined by the values of the synthesized support degree. The aforementioned optimal sensor placement methods are mostly used for modal identification, the optimal sensor placement methods were also additional studied with the aim to the structural response reconstruction and monitoring [11], such like the optimal placement of sensors for sub-surface fatigue crack monitoring [12], the sensor positioning and choice of the number of sensors were optimized in terms of the reconstruction on the temperature field considering the error propagation in case of uncertain measurements [13], the optimal sensor placement for enhancing sensitivity to change in stiffness was proposed to find the optimal configuration of sensors that would best predict structural damage [14], the optimal sensor placement methodology was proposed so as to better estimate the vibration response of the entire structure [15] and so on. The stress distribution and displacement development of the structure are important monitoring parameters for structural safety estimation and should be considered in the optimal sensor placement method for such strain or displacement sensors. For such consideration, the measurements of strain or displacement sensors can not only display the structural responses at the placement where the sensors are located but also reflect the structural responses at placements correlated with sensor locations. An additional objective of optimal sensor placement for strain or displacement sensors is to avoid redundancy among the measurements from different measuring points. Correlation is a parameter that can represent the redundancy among the measurements; higher (lower) correlations can lead to greater (less) redundancy, i.e., more (less) overlapping information was measured. Therefore, selecting measuring points with little association with each other can make it possible to gain more independent measuring information that is more comprehensive and reliable.

To reduce the redundancy information of measurement sensor systems and maximize the function of limited sensors, as well as to evaluate the structural responses with the measurements from limited sensors, this paper presents a method of optimal sensor placement based on correlation. First, a correlation matrix of potential sensor placements is established and processed into a binary matrix. Second, the potential sensor placements are preliminarily optimized and classified using the bond energy algorithm (BEA), in which the potential sensor placements in the same category are strongly correlated and the potential sensor placements in different categories are weakly correlated. Then, the final position and number of measuring points can be determined according to the proposed principle of optimal sensor placement. The optimal sensor placement in the deformation monitoring of a single-layer Schwedler reticulated dome-like steel structure is conducted to verify the effectiveness of the proposed method by analyzing the information entropy of the optimized results.

2. Data correlation analysis of structural responses

2.1. The establishment of the correlation matrix

The correlation degree is usually expressed as γ, which describes the association of structure response information between two positions and reflects their correlation level. It is a non-dimensional parameter that ranges from -1.0 to 1.0 (including -1.0 and 1.0). The closer the absolute correlation value is to 1, the more strongly the structural responses from two positions are correlated, whereas the closer the absolute correlation value is to 0, the more weakly the structural responses from the two positions are correlated.

It is assumed that $X \in R^{m \times n}$ represents the response information matrix of n positions in m load cases, i.e.,

$$X = [x_1 \quad x_2 \quad \cdots \quad x_n] = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \tag{1}$$

For two arbitrary positions at i and j, the response information is x_i and x_j, and the sample variance and covariance computation formulas are as follows:

$$Var(x_i) = \frac{1}{m-1}\sum_{k=1}^{m}(x_{ki} - \overline{x_i})^2 = \sigma_{ii} \tag{2}$$

$$Var(x_j) = \frac{1}{m-1}\sum_{k=1}^{m}(x_{kj} - \overline{x_j})^2 = \sigma_{jj} \tag{3}$$

$$Cov(x_i, x_j) = \frac{1}{m-1}\sum_{k=1}^{m}(x_{ki} - \overline{x_i})(x_{kj} - \overline{x_j}) = \sigma_{ij} \tag{4}$$

where $\overline{x_i} = \frac{1}{n}\sum_{k=1}^{n} x_{ki}$ and $\overline{x_j} = \frac{1}{n}\sum_{k=1}^{n} x_{kj}$ are the mean values of x_i and x_j, respectively, and the correlation degree formula of structural response information at positions i and j is:

$$\gamma_{ij} = Corr(x_i, x_j) = \frac{Cov(x_i, x_j)}{\sqrt{Var(x_i) * Var(x_j)}} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii} * \sigma_{jj}}} \tag{5}$$

The correlation matrix R can be expressed as

$$R = [\gamma_{ij}]_{n \times n} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \cdots & \gamma_{nn} \end{bmatrix} \tag{6}$$

2.2. Binary processing to correlation matrix

The correlation threshold is expressed as η; when the absolute value of the correlation is no smaller than the correlation threshold, i.e., $|r_{ij}| \geqslant \eta$, the element in the ith row and jth column of the correlation matrix is replaced by 1, and when the absolute value of correlation is smaller than η, i.e., $|r_{ij}| < \eta$, the element in the ith row and jth column of the correlation matrix is replaced by 0. With this binary processing of the correlation matrix, the equivalent correlation matrix D, whose elements are all one or zero, can be expressed as

$$D = [d_{ij}]_{n \times n} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & \cdots & d_{nn} \end{bmatrix} \tag{7}$$

3. Bond energy algorithm for equivalent correlation matrix

3.1. Matrix transformation using the bond energy algorithm

The bond energy algorithm (BEA) [16] is a type of clustering approach, which can transform the matrix into submatrices with characteristics of block division by calculating the bond energy of elements into rows and columns. The BEA is used to calculate the bond energy of the rows and columns in the equivalent correlation matrix. The row and column transformation is taken subsequently based on the value of the bond energy to enable an equivalent correlation matrix with the characteristics of block division. The equivalent correlation matrix can be represented with several diagonal