Measurement 69 (2015) 9–19

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Study on Single-bin Sliding DFT algorithms: Comparison, stability issues and frequency adaptivity

Carlos M. Orallo [*,1], Ignacio Carugati [1], Patricio G. Donato [1], Sebastian Maestri [1]

Universidad Nacional de Mar del Plata, Facultad de Ingeniería, Laboratorio de Instrumentación y Control, Juan B. Justo 4302, Mar del Plata, Argentina

ARTICLE INFO

Article history:
Received 20 October 2014
Received in revised form 9 February 2015
Accepted 10 March 2015
Available online 19 March 2015

Keywords:
Power quality
Harmonics measurement
System modeling
Discrete Fourier transform
Digital signal processing
Control design

ABSTRACT

The standard method for spectrum analysis is the Discrete Fourier Transform(DFT), typically implemented using a Fast Fourier Transform (FFT) algorithm. However, certain applications require an on-line spectrum analysis only on a subset of M frequencies of an N-point DFT ($M < N$). In such cases, the use of Single-bin Sliding DFT (Sb-SDFT) is preferred over the direct application of FFT. Along these lines, the most popular algorithms are the Sliding Discrete Fourier Transform (SDFT), the Sliding Goertzel Transform (SGT), the Modulated Sliding Discrete Fourier Transform (mSDFT), and the S. Douglas and J. Soh algorithm (D&S). Even though these methods seem to differ, they are derived from the conventional DFT using distinct approaches and properties. To better understand the advantages, limitations and similarities each of them have, this work thoroughly evaluates and compares the four Sb-SDFT methods. What is more, the direct application of these Sb-SDFTs may lead to inaccuracies due to spectral leakage and picket-fence effects, common pitfalls inherited by every DFT-based method. For this reason, a unified model of the Sb-SDFT methods is proposed, whose aim is to design a frequency adaptive control loop. This frequency adaptability allows to mitigate the problems associated with improper sampling frequency. By using this unified model, the election of the Sb-SDFT algorithm is independent of the controller design and all the methods are equivalent. Theoretical results are validated by simulations and a DSP implementation of the four frequency adaptive Single-bin Sliding DFT methods.

1. Introduction

The proliferation of advanced power electronic technologies, such as switching power supplies and adjustable speed motor drives, among others, have led to an increment in the harmonic currents injected into power systems, causing power quality degradation. Harmonic pollution can cause serious problems in power systems, e.g., it can accentuate losses in distribution networks and rotating machines, lead to inaccurate operation of protection and control systems, damage sensitive loads and create significant interference in communication systems. In addition, the system frequency may deviate from its nominal value due to the imbalance between power generation and load demand. Therefore, interest lies in measuring the harmonic components of non-stationary signals, such as grid voltage and currents for grid monitoring and implementation of preventive strategies [1–3].

Various approaches estimate the harmonic content of electrical signals. The method most commonly used is the Discrete Fourier Transform (DFT), implemented by the Fast Fourier Transform (FFT) [4–6] due to its computational efficiency. By transforming the measured signal

* Corresponding author.
E-mail address: orallo@fi.mdp.edu.ar (C.M. Orallo).
[1] The authors are with the Universidad Nacional de Mar del Plata, CONICET.

http://dx.doi.org/10.1016/j.measurement.2015.03.011

from the time domain to the frequency domain, FFT can accurately track its harmonic components.

For some real-time applications, the direct application of conventional DFT methods or more efficient FFT techniques can be complex and/or involve an excessive computational cost. However, certain applications require an on-line spectrum analysis only over a subset of M frequencies of an N-point DFT ($M < N$). For this scenario, the common practice is to utilize Single-bin Sliding DFT algorithms (Sb-SDFT). These algorithms efficiently calculate a unique spectral component of an N-point DFT. The Sliding Discrete Fourier Transform (SDFT) [7,8], the Sliding Goertzel Transform (SGT) [9,10], the Modulated Sliding Discrete Fourier Transform (mSDFT) [11] and the S. Douglas and J. Soh algorithm (D&S) [12] are among the most popular Sb-SDFT methods. These four techniques have spectral bin output rates equal to the input data rate on a sample-by-sample basis.

Nonetheless, since the harmonic level and the fundamental frequency in the power system are usually time-varying, the direct application of DFT-based methods for spectral analysis may lead to inaccuracies due to spectral leakage and picket-fence effects [13]. These unwanted effects are related to the frequency variation and improperly selected sampling time window. In order to solve this problem, the authors present in [14] a harmonic measurement method which employs the mSDFT algorithm and a frequency adaptive mechanism. This mechanism, named Variable Sampling Period Technique (VSPT), dynamically adjusts the sampling period to exactly N times the fundamental frequency, thereby avoiding the above-mentioned problems.

Despite the fact that Sb-SDFT methods seem to be different, they are derived from the conventional DFT with distinct approaches and properties. Thus they bear great similarities that are frequently overlooked. This paper evaluates and compares the four selected Sb-SDFT algorithms in diverse operational conditions, identifying the similarities between them. Based on this analysis, a unified mathematical model is proposed for the implementation of VSPT in order to achieve a frequency adaptive system. The purpose of obtaining a unified model is to make the choice of an Sb-SDFT independent from the design of the control in charge of frequency adjustment, rendering the design and implementation process more flexible and convenient. Moreover, it allows to change the algorithm without redesigning the control loop. By using the unified model proposed here, the benefits of this frequency adaptive mechanism can be extrapolated to all the algorithms discussed in this manuscript. The model obtained, together with its usefulness in the design process, is validated by experimental results obtained in a DSP platform.

The paper is organized as follows: Section 2 presents a brief review of Sb-SDFT. The steady-state characteristics of the reviewed methods are analyzed in Section 3, while their dynamic behaviors are presented in Section 4. In Section 5, a scheme for frequency adaptation (VSPT) to mitigate the inaccuracies resulting from the spectral leakage and picket-fence effect is introduced. A unified model is presented to generalize this scheme to all Sb-SDFT along with simulation results. The experimental results for the implementation of the reviewed Sb-SDFT with VSPT, based on the unified model, are shown in Section 6. Finally, the conclusions of this work are drawn in Section 7.

2. Review of Single-bin Sliding Discrete Fourier Transforms

The standard method for spectrum analysis in digital signal processing is the Discrete Fourier Transform (DFT). DFT converts a finite series of equally spaced samples of a function into a series of coefficients of a finite combination of complex sinusoids, ordered by their frequencies. Therefore, DFT converts the sampled function from its original time domain to the frequency domain.

The DFT of the sequence $x(n)$ is defined as

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{-kn} \tag{1}$$

where $X(k)$ is the DFT output coefficient, $W_N = e^{j2\pi/N}$ is the complex twiddle factor, N is the sequence length, k is the frequency domain index ($0 \leqslant k \leqslant N-1$) and n is the time domain index [4].

For a number of real-time applications, the direct application of conventional DFT methods can be complex and demand high computational effort. Additionally, there are applications that require spectrum analysis only over a subset of M frequencies of an N-point DFT. In such instances, it is convenient to use a Single-bin Sliding Discrete Fourier Transform (Sb-SDFT) algorithm, which computes a single complex DFT spectral bin value by means of a sliding window.

One of the most common Sb-SDFTs, is the Sliding Discrete Fourier Transform (SDFT). SDFT is a recursive algorithm that performs an N-point DFT on time samples within a sliding window on a sample-by-sample basis. The time window is advanced one sample at a time, and a new N-point DFT is calculated. The principle used for SDFT is known as the DFT shifting theorem, or the circular shift property.

SDFT can be recursively implemented to calculate Eq. (1) for a desired k-bin, as:

$$X_k(n) = W_N^k X_k(n-1) - x(n-N) + x(n) \tag{2}$$

where $X_k(n)$ is calculated by phase shifting the sum of the previous $X_k(n-1)$ with the difference between the current and delayed input sample, $x(n)$ and $x(n-N)$, respectively [7,8].

SDFT is computationally efficient, as it only requires one (complex) multiplication and two additions per time instant. Nevertheless, the implementation of Eq. (2) as an IIR filter in a system with finite word-length precision brings about a rounding error in the implementation of the W_N^k coefficient, resulting in accumulated errors and potential instabilities. The latter is explained by wrong cancellations between poles and zeroes as well as by poles displacement outside the unit circle [15]. To achieve stability, a damping factor (r) must be included to force the poles and zeroes to be at a radius of r inside the unit circle. Then, the intrinsically stable version of the SDFT is: