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Performance measurement of decision-making units under uncertainty conditions: An approach based on double frontier analysis

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ABSTRACT

Data envelopment analysis (DEA) is an approach to measure the relative efficiency of a set of decision-making units (DMUs) which uses multiple inputs to produce multiple outputs. In real world situations, due to uncertainty, DEA is sometimes faced with imprecise inputs and/or outputs. Therefore, performance measurement must often be performed under uncertainty conditions. Generally, the performance of DMUs can be evaluated from two perspectives—optimistic and pessimistic. As a result, two different evaluations are obtained for each DMU. In this article, we first obtain the efficiencies of the DMUs under evaluation from both optimistic and pessimistic views. The optimistic view evaluates each DMU with a set of the most desirable weights; the efficiencies measured by the optimistic approach are called optimistic efficiencies. The pessimistic view evaluates each DMU with a set of the most undesirable weights; the efficiencies measured by the pessimistic approach are called pessimistic efficiencies. Then it is shown that the outcomes of these two evaluations are conflicting with each other, being undoubtedly biased, unrealistic, and unconvincing. To overcome this problem, we propose a new measure of overall performance which is used for integrating the measures obtained from optimistic and pessimistic views and we will use it to identify the DMU with the best performance under uncertainty conditions. Also, we propose new fuzzy DEA models that evaluate a DMU from the pessimistic perspective in a fuzzy context. The proposed measure will be shown with two numerical examples, including the selection of a flexible manufacturing system.

1. Introduction

Data envelopment analysis (DEA) was developed twenty years after the initial work of Farrell [39] and based on his ideas by Charnes et al. [19]. It is a non-parametric method to measure the performance of a group of decision-making units (DMUs) which consume multiple inputs and produce multiple outputs. This method has been widely used for measuring and benchmarking relative efficiency of many decision-making entities in the public and private sectors. In recent years, many articles and reports have studied the DEA application in educational systems, banking, agriculture, health care units, national and international comparison, power industry, military logistics, transportation, insurance, and so on [3,15,16,28,46,50,57,61,62,70,89,112].

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In classical DEA models [17,19], the efficiency of a DMU is obtained by maximizing the ratio of the weighted sum of its outputs to the weighted sum of its inputs, provided that this ratio does not exceed 1 for each DMU. In other words, for each DMU under evaluation, one can evaluate its efficiency with input and output weights that are most desirable only for themselves. Thus, it is said that the approaches proposed by Charnes et al. [19], and Banker et al. [17] measure the performances of DMUs from the optimistic view, and the efficiencies measured in this approach are called the *best relative efficiency* or *optimistic efficiency* and its measure in the input-oriented mode, is limited to values less than or equal to one. If the optimistic efficiency measure of a DMU is equal to one, it is called DEA efficient or optimistic efficient; otherwise, it is called DEA non-efficient or optimistic non-efficient. It is commonly believed that the performance of optimistic efficient DMUs is better than optimistic non-efficient ones.

On the other hand, an approach is proposed that measures the performances of DMUs from the pessimistic view [49,63,71,72]. In this approach, each DMU under evaluation can evaluate its efficiency with input and output weights that are most undesirable only for themselves. Thus, the efficiencies measured from the pessimistic view are called the *worst relative efficiency* or *pessimistic efficiency* and its measure in the input-oriented mode, is limited to values greater than or equal to one. If the pessimistic efficiency measure of a DMU is equal to one, it is called pessimistic inefficient or DEA inefficient; otherwise, it is called pessimistic non-inefficient or DEA non-inefficient. It is commonly believed that the performance of pessimistic inefficient DMUs is worse than pessimistic non-inefficient ones.

Optimistic and pessimistic efficiencies measure two extremes of each DMU performance. Any evaluation approach that considers only one of them suffers from bias. To determine the overall performance of each DMU, they both should be considered simultaneously. An approach that evaluates the performance of each DMU for both optimistic and pessimistic efficiencies is called “double frontier analysis” approach.

The traditional DEA models assume that exact data are available for all inputs and outputs. In some applications, however, some of the factors may include imprecise data [4,58,59,77]. The nature of these imprecise data depends on the characteristics of the particular problem [56,57,65]. For example, they could be in the form of missing values, integer values, judgment data, fuzzy data, rank data, etc. [5,26,27,29,54,55]. Various DEA models have been developed for dealing with imprecise data [8,53,66,67,77]. We use the DEA models proposed by Wang et al. [97], Wang et al. [101] for obtaining the best interval and fuzzy efficiencies of the DMUs. Besides, we obtain the pessimistic efficiency intervals of the DMUs using the models proposed by Azizi and Ganjeh Ajirlu [11]. Then, due to the need for developing the fuzzy DEA theory and methodology, as well as its real-world applications, we propose new fuzzy DEA models that evaluate a DMU from the pessimistic perspective in a fuzzy context. Finally, using the double frontier analysis approach, we propose a new measure for evaluating the performance of DMUs. Two numerical examples are presented to illustrate the application of the proposed fuzzy DEA models and performance measure.

Intuitively, the efficiency scores calculated from imprecise data should also be imprecise. Thus, the final efficiency score of each DMU with interval input and output data is determined by an interval number, and with fuzzy input and output data is determined by a fuzzy number. In order to compare and rank interval and fuzzy numbers, several approaches have been developed [38,42,48,60,73,74,79,90,105,106]. We use an approach based on the preference degree for comparing and ranking interval and fuzzy efficiencies of DMUs [101,104].

This article is organized as follows. Section 2 reviews the literature on the double frontier analysis approach. Section 3 introduces imprecise DEA models for measuring optimistic and pessimistic efficiencies of DMUs. Section 4 analyzes the DEA models proposed by Entani et al. [34]. Section 5 introduces overall performance measures. Numerical examples are presented in Section 6. Finally, Section 7 is the conclusion.

2. Literature review

Since the performances of DMUs can be measured from both optimistic and pessimistic views, two efficiencies are obtained for each DMU: optimistic efficiency and pessimistic efficiency. These two efficiency scores are usually considered with two different methods in the overall evaluation: (i) as an efficiency interval (i.e. the efficiency of a DMU is an interval between optimistic and pessimistic values); and (ii) integration of optimistic and pessimistic efficiencies (which is defined by averaging or other mathematical relations between the two measures).

This section first reviews the articles relating to case (i) and then case (ii).

Among the first people who studied the performances of DMUs from both optimistic and pessimistic views, one can point to Doyle et al. [31] and Entani et al. [34]. Their proposed models have a similar structure. In their study, the obtained efficiency values constitute an efficiency interval whose lower bound is the pessimistic efficiency and upper bound is the optimistic efficiency. Entani et al. [34] proposed two DEA models to obtain the efficiency interval for each DMU. Their proposed pair DEA model was developed initially for crisp data and then was extended to interval and fuzzy data. In theory, their proposed DEA model could work with interval data and fuzzy data but it has important drawbacks. It only uses data for one input and one output to determine the lower bound of the efficiency interval for each DMU, regardless of the number of inputs and outputs in the model. It leads to the loss of information about the input and output data of the DMU under evaluation. Moreover, it uses variable production frontiers to measure efficiency intervals for DMUs with input and/or output interval data. To overcome the drawbacks of the DEA models proposed by Entani et al. [34], Wang and Yang [103] provided a pair of bounded DEA models to work with crisp data. It makes the most use of all input and output data when measuring performance and measures the optimistic and