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Applying particle swarm optimization algorithm to roundness error evaluation based on minimum zone circle

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ABSTRACT

Minimum zone circle (MZC) method and least square circle (LSC) method are two most commonly used methods to evaluate roundness, but only the MZC method complies with the standard definition and can obtain the minimum roundness error value. The determination of the center of MZC is a nonlinear optimization problem which is suitable to be solved by particle swarm optimization (PSO) algorithms. In this paper, the standard PSO algorithm was introduced and theory analysis about the impact of value selection of some important parameters, such as inertia weight ω, on the algorithm's stability and convergence was carried on so as to provide basis for giving these parameters better values. Furthermore, the superiority of making ω decrease linearly with iterations was verified through a computation experiment in terms of stability and accuracy, compared with the other three cases of $\omega = 1, 0.5, 0$. Based on the analysis, the novel PSO algorithm, with ω decreasing linearly from 0.9 to 0.4 and the LSC center as the initial positions of the particles, is implemented to obtain MZC-based roundness errors of sampling points collected from circular section profiles by a coordinate measuring machine (CMM). By comparing the novel PSO–MZC results with the LSC-based results, it is concluded that the former are a little smaller than the latter, which verifies that the novel PSO algorithm is feasible to calculate roundness error and the fact that a LSC-based one is generally larger than a MZC-based result; the values of the two roundness errors are both related to sample size and increase with an increase in the sample size with a decreasing increment.

1. Introduction

Roundness error indicates the variation between the actual circle and the ideal one of a cross-section profile of a rotational part. When the variation is within the allowable region, the part meets roundness requirements which include not only functional needs, i.e. the suitability for assembly with its designed counterpart(s) and the proper functioning of a mechanical system, but also issues such as manufacturability, esthetics, and conformance to regulations [1]. So, it is required to develop an automatic evaluation method for roundness error.

Form tolerances of a component with reference to an ideal geometric feature are defined in [2]. As for roundness error, the reference features (circles), mainly consisting of the minimum circumscribed circle (MCC), the maximum inscribed circle (MIC), MZC and LSC, have been established to obtain roundness error. Among the four methods, only the MZC complies with the standards [2,3] and has the minimum value, which was verified in Ref. [4,5]. The LSC method is based on sound mathematical principles that minimize the sum of the squared deviations of the measured points from the fitted circle [6,7], and is generally adopted by CMMs. The LSC method is robust and efficient

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in computation [8], but does not follow the standard definition intently. The values of roundness error determined by the LSC method are generally larger than the actual ones, which may lead to rejection of good parts [5]. The MIC and MCC can be used when mating is involved. Gadelmawla [5] introduced several simple and efficient algorithms to evaluate the roundness error using MCC, MIC, MZC, and the results showed the roundness errors determined by the MCC and MIC methods are larger than that by the MZC method and the evaluations by using the two methods are more time-consuming than that of the MZC method with the same solution algorithms.

Roundness error of the MZC method is the radius difference of two concentric circles with the minimum radial separation, within the area between which all measurement points of the profile should lie [2,3]. The determination of MZC-based round error is a two-dimensional optimization problem, the objective of which is to find an optimum circle center of the two concentric circles. Some intelligent optimization algorithms, more efficient than computational geometry techniques in obtaining the minimum-zone solution [5,9], have been developed and proved able to provide robust and optimal solution, such as methods based on PSO algorithm [10–12], genetic algorithm (GA) [13,14]. Cui et al. [10] used a PSO algorithm to obtain MZC-based roundness errors, and the results revealed the solving process of the PSO had a faster convergence than that of GA. Sun [11] proposed a machine vision-based roundness measurement method that applied the standard PSO algorithm with a constant inertia weight to obtain MIC, MCC and MZC results, and the experimental results revealed that the PSO algorithm effectively solved the MIC, MCC, and MZC problems and outperformed GA in terms of both accuracy and efficiency.

Given that there are so many researches on obtaining MZC-based roundness errors by using GA, which are well developed, and the approaches of using PSO algorithm are feasible and suitable, in this paper, a novel PSO algorithm was adopted to solve MZC-based roundness errors. The novel PSO algorithm is a result of changing the constant inertia weight of the standard PSO presented in [11] into a linearly decreasing one with the PSO iterations and the superiority of the dispose is verified through a computation experiments in Section 4. In addition, the standard PSO algorithm with inertia weights is introduced and theory analysis on the parameter selection and convergence of the PSO algorithm is made in Section 2; in Section 3, the MZC-based roundness error solution is formulated and the novel PSO algorithm is implemented to give the roundness error using the coordinate data collected by a CMM.

2. PSO algorithm

2.1. PSO algorithm principle

The simple PSO algorithm, first introduced in 1995 by Eberhart and Kennedy [15], is an evolutionary computation technique, developed for solving optimization problems of continuous non-linear, constrained and unconstrained, non differentiable multi-modal functions. In a PSO algorithm, each candidate solution, called a particle, is treated as a position in the n-dimensional solution space. "Particle swarm" is represented by $\mathbf{X} = (\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_m)^T$ where m is the swarm size and the ith particle is represented as $\mathbf{X}_i = (x_{i1}, x_{i2}, \ldots, x_{in})$. Each particle has a velocity $\mathbf{V}_i = (v_{i1}, v_{i2}, \ldots, v_{in})$ which decides the flying distance and direction, and also has its fitness value which is determined by the target optimization function F with the positions of the particles as input values. In the PSO algorithm used in this paper, the position and velocity of each particle are adjusted each iteration [16–18], expressed by

$$\begin{cases} \mathbf{V}_i^{k+1} = \omega \times \mathbf{V}_i^k + c_1 \times r_1^k \times \left(\mathbf{P}_i^k - \mathbf{X}_i^k\right) \\ \qquad +c_2 \times r_2^k \times \left(\mathbf{P}_g^k - \mathbf{X}_i^k\right), \qquad i = 1, 2, \ldots, m, \\ \mathbf{X}_i^{k+1} = \mathbf{X}_i^k + \mathbf{V}_i^{k+1} \end{cases} \tag{1}$$

where k is the iteration number; i is particle number; c_1, c_2 are the learning factors used to adjust the flying distances of the particles from the individual optimal value and from the global optimal value, respectively; r_1, r_2 are two random numbers in the range of [0,1]; ω is the inertia weight, first introduced by Shi and Eberhart [16] into the simple PSO in order to improve the convergence of the simple PSO.

In addition, the fitness function $F(\mathbf{X})$ is evaluated with a new position of the particle swarm. The algorithm in pseudocode is listed as follows:

```
{
Initialize the swarm: X_i, V_i
For k = 1: Miter
  For i = 1 to m
    If F(X_i) < F(P_i) then (P_i = X_i)
    If F(P_i) < F(P_g) then (P_g = P_i)
  End
  Update each particle according to Eq. (1)
End
}
```

When a particle discovers a position $\mathbf{X}_i^k$ that is better than what it has found previously, represented by $\mathbf{P}_i = (p_{i1}, p_{i2}, \ldots, p_{in})$, it updates $\mathbf{P}_i$ using the new position. The difference between $\mathbf{P}_i$ and the individual's new position $\mathbf{X}_i^k$ is stochastically added to the current velocity $\mathbf{V}_i^k$, causing the trajectory to oscillate around $\mathbf{P}_i$. Further, each particle is defined within the context of a topological neighborhood comprising itself and other particles in the swarm. The stochastically weighted difference between the swarm's best position represented by $\mathbf{P}_g = (p_{g1}, p_{g2}, \ldots, p_{gn})$, and $\mathbf{X}_i^k$ is also added to its velocity $\mathbf{V}_i^k$, which is prepared for the next iteration step. These adjustments to the particle's movement through the space cause the particle to search around the two best positions. Often when the number of iterations reaches the maximum iteration number *Miter*, the algorithm routine is terminated.

The PSO is mainly characterized by using few components to obtain excellent-quality solutions. The real-life application of PSO is widespread recently, such as har-