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Measurement and evaluation of the apparent modulus of elasticity of apple based on Hooke's, Hertz's and Boussinesq's theories

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ABSTRACT

Awareness on the mechanical properties of agricultural products is necessary in order to estimate and predict the deformation of viscoelastic materials under external loads for system design, transport, processing and packaging. The aim of this study was to evaluate three theories (i.e. Hooke, Hertz and Boussinesq) on the apparent modulus of elasticity of some apple varieties i.e. Golden Delicious, Red Delicious and Granny Smith. The theoretical results were analyzed in a factorial experiment with completely randomized design with 9 treatments and 15 replicates. The results showed the practical usability of Hertz's theory with better prediction accuracy whilst the Boussinesq's theory showed a significant difference of predicted modulus of elasticity compared to other theories and the values reported in some publications. Although, the Hook's theory enables the identification of a bio-yield point on its force–deformation curve, but Hertz's theory was recommended as the most appropriate method due to easiness of working on a cylindrical sample of apple and the closer agreement with reality. Based on the results of this study, Golden Delicious and Granny Smith varieties had the lowest and the highest modulus of elasticity as 2.211 MPa and 3.431 MPa, respectively. This difference shows the firmness of apple varieties and different tissue responses to external loads and forces accordingly.

1. Introduction

In general, knowledge about the mechanical properties of agricultural products is required for proper design of processing machines. Designers of such machines, before any decide about fabrication and application, should examine and understand the mechanical properties of the products. According to the bibliography, many researchers have investigated mechanical properties of different agricultural products such as fruits, vegetables, and grains [1]. Meanwhile, determination of modulus of elasticity as one of the most important mechanical properties of agricultural products representing tissue stiffness is of great interest to food engineers because this parameter together with other mechanical properties e.g. compression failure stress, tension failure stress, failure energy, firmness, Poisson's ratio and toughness are far important in machinery design and improvement [1]. The apparent modulus of elasticity is defined as the ratio of stress to strain within the elastic range. It is called apparent because the viscoelastic properties of agricultural materials are assumed to be elastic [2]. Some reports have documented the examination of several mechanical parameters of apple tissues such as the apparent modulus of elasticity, Poisson's ratio and Lam's coefficient based on the Hertz's theory [3–5]. Based on these reports, the apparent modulus of elasticity,

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Nomenclature

E	apparent modulus of elasticity (MPa)
ΔL	length difference before and after test (mm)
A	initial cross section area of the specimen (mm^2)
L	initial length of the specimen (mm)
F	elastic limit force (N)
d	diameter of curvature of the spherical indenter (mm)
$MCw{\cdot}b$	average moisture content of fruits and vegetables
R_U	minimum radii of curvature of the sample at the upper point of contact (mm)
R_L	maximum radii of curvature of the sample at the upper point of contact (mm)
R'_U	minimum radii of curvature of the sample at the lower point of contact (mm)
R'_L	maximum radii of curvature of the sample at the lower point of contact (mm)
D	deformation (mm)
a	radii of the rigid die (mm)
μ	Poisson's ratio (dimensionless)

Lam's coefficient, and Poisson's ratio changed as a function of degree of maturity and storage time. Also, some researchers have used Hook's theory in order to study the mechanical properties of apple fruit including modulus of elasticity, failure stress, failure strain and failure energy by placing the cylindrical specimens of apple tissue under uniaxial compression test [6–11]. To date, no report has been published for the determination of modulus of elasticity of apple considering different well known theories. Therefore, the aim of this study was to measure and compare the apparent modulus of elasticity of three export varieties of apple estimated by Hooke's, Hertz's and Boussinesq's theories under uniaxial compression tests.

2. Materials and methods

In this study, three export varieties of apple namely Golden Delicious, Red Delicious and Granny Smith were examined. The samples were harvested and transported to the laboratory of Shahrekord University immediately after harvest and all measurements were performed within 48 h after harvest. The average of MCw.b. for Golden Delicious, Red Delicious and Granny Smith were 81.91%, 82.83% and 84.41%, respectively. In order to investigate the effect of Hooke's, Hertz's and Boussinesq's theories for estimation of the apparent modulus of elasticity for apples tissue, factorial experiment in a completely randomized design with 9 treatments in 15 replicates were used. Treatments were a combination of the factors including apple varieties in three levels (Golden Delicious, Red Delicious and Granny Smith) and three levels of theories (Hooke, Hertz and Boussinesq). All experiments were performed within a temperature range of 20–22 °C.

2.1. Hooke's theory

Based on Hook's theory, a cylindrical or cubic sample from the homogenous domestic tissue of the product is prepared and compressed uniaxially between two parallel flat plates to a certain degree of deformation (e.g. rupture). The cylindrical samples were usually within 10–20 mm diameter and 20–30 mm height [2,6]. The apparent modulus of elasticity is the slope of the initial linear part of the stress–strain curve of uniaxial compression as given in Eq. (1)

$$E = \frac{\sigma}{\varepsilon} = \frac{FL}{A\Delta L} \tag{1}$$

According to the ASAE standard S368.4, the upper limit of elastic range for determining the apparent modulus is selected at the point of inflection, PI (i.e. the point at which the rate of change of slope (second derivative) of the curve becomes zero). The point of inflection associated with the change in slope suggests that some types of failure are beginning. In this study, the compression tests were accomplished using a universal Instron machine (Santam, STM-20, Tehran, IRAN) on the cylindrical samples taken from the tissue of the studied varieties of apple with 15 replicates for each variety. The samples were taken from the equatorial line of the apples as some previous studies reported no significant difference of mechanical properties of the samples taken from the equatorial line of apples [6,12]. Samples were taken using a steel sharp cylinder with a diameter of 10 mm and height of 15 mm [13]. For each variety, apparent modulus was calculated as observed with some initial tests (Fig. 1a). As illustrated in Fig. 1a, the biological yield point in agricultural products is a point on force–deformation curve where an increase in deformation results in a decrease or no change in force [2,14]. For each variety, after obtaining the elastic limit from the initial tests, cylindrical samples were compressed under axial loading for 8 mm as shown in Fig. 1b [6]. Loading rate for all tests was 25.4 mm/min [14]. Test was then stopped and for each sample, force–deformation curve was plotted by the software installed (Fig. 1a).

2.2. Hertz's theory

Based on the equations of the Hertz's theory, the values of contact area of two bodies (A), maximum contact stress (σ_{max}), deformations of the two bodies at the point of contact (D), and modulus of elasticity of material (E) can be calculated. To achieve a mathematical solution for the Hertz's theory, some initial assumptions were made according to Kosma and Cunningham [15].

In order to calculate the apparent modulus of elasticity of apple samples according to Hertz's theory, with assumption of small deformation and homogeneous elastic mate-