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A modified Semi-Oriented Radial Measure for target setting with negative data

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ABSTRACT

Over the last few years Data Envelopment Analysis (DEA) has been gaining increasing popularity as a tool for measuring efficiency and productivity of Decision Making Units (DMUs). Conventional DEA models assume non-negative inputs and outputs. However, in many real applications, some inputs and/or outputs can take negative values. Recently, Emrouznejad et al. [6] introduced a Semi-Oriented Radial Measure (SORM) for modelling DEA with negative data. This paper points out some issues in target setting with SORM models and introduces a modified SORM approach. An empirical study in bank sector demonstrates the applicability of the proposed model.

1. Introduction

Data Envelopment Analysis (DEA) is a mathematical programming approach that repeatedly used for measuring performance of Decision Making Units (DMUs) that convert multiple inputs into multiple outputs [2]. DEA has proven to be an excellent tool in decision support system, for measuring efficiency and productivity of DMUs. Many applications of DEA are reported in the literature e.g., [9,15,22,17]. For full set of applications of DEA and a road map of how to use DEA see Emrouznejad et al. [5]. One of the main drawbacks in the conventional DEA models is that they accept only non-negative input and output values. However, there are many occasions in which some inputs and/or outputs may take negative values, for example see [18,16,20].

Recent years, DEA models with negative data have been successfully used in several applications: banking sector [10], productivity assessment [23], super-efficiency [11], and Diabat et al. [4], are just few examples in this regard.

Recently, Emrouznejad et al. [6,7] have explored negative data in DEA; including a brief review of the recent approaches in this topic by Portela et al. [19] and Sharp et al. [21] which are referred as Range Directional Measure (RDM) and Modified Slack Based Measure (MSBM), respectively. Emrouznejad et al. [6] used a partitioning approach in modelling negative data, hence, they proposed a Semi-Oriented Radial Measure (SORM) for performance evaluation of the observed production units.

The SORM model is suggested to handle the cases where some DMUs have positive and other negative values on a variable and also when DMUs have negative input(s) and negative output(s) at the same time. The key idea of this approach is to replace an input/output variable which can take positive values for some and negative values for other units by a difference of two non-negative variables, which the first element of this substitution includes only

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positive values and the second element includes the absolute values of the negative part.

Their approach yields a new production technology, in which the absolute values of negative outputs are considered as inputs and on the other side, the absolute values of negative inputs are considered as outputs.

Similar to RDM and MSBM, SORM is also one of the most commonly used approaches in dealing with negative data in recent DEA literature in which dataset includes variables with both positive and negative values. Kazemi Matin and Azizi [14] and afterward Kazemi Matin and Izadi [13] introduced procedures for setting targets and computing efficiency scores in DEA with negative data, including decision maker interest to a specific level of input/output variables in benchmarks. The SORM results are used in their applications for comparison purposes and illustrations. Recently, Cheng et al. [3] proposed a variant of radial measure (VRM) to handle variables with both positive and negative data in DMUs. They provide a review of existing approaches in DEA, including RDM and SORM, and show VRM is a unit invariant model and can deal with all cases of the presence of negative data.

One key issue that makes SORM model different from other methods such as RDM and MSBM is that it converts each input–output variable essentially as the sum of two variables, one taking its negative value and the other its positive value. The advantage of this representation is that the negative part of a variable can be dealt with in absolute value terms and thus in positive format without arbitrary changes of origin as might otherwise be necessary to achieve positive values.

The current paper primarily concerned with SORM approaches from both theoretical foundation and efficiency measurement modelling. We show that dealing with negative outputs as inputs is inconsistent with production process assumptions. Furthermore, we will show that the standard SORM has some inherent limitations to satisfy a classical efficiency measure that under specific situation may fail to distinguish between efficient and inefficient units. We will also see that the radial direction of SORM, in which the performance is measured, can itself prevent to suggest target points with positive values for negative components. Afterward, we will introduce a new model for inefficiency evaluation and targets setting in a modified SORM approach.

The rest of this paper is organized as follows. Section 2 gives a brief explanation of the SORM model and highlights some issues in efficiency measurement and target setting using SORM. A modified SORM model has been introduced in Section 3. Section 4 is devoted to an illustration application. Conclusion and further remarks are given in Section 5.

2. Modelling negative data in SORM

In DEA, it is most common to characterize each observed DMU by a pair of non-negative valued vectors $(\boldsymbol{x}_j, \boldsymbol{y}_j) \in \Re_+^{m+s},\ j \in J = \{1, \ldots, n\}$ in which the input vector $\boldsymbol{x}_j = (x_{1j}, \ldots, x_{mj})$ is consumed to produce the output vector $\boldsymbol{y}_j = (y_{ij}, \ldots, y_{sj})$. The classic BCC DEA model by Banker et al. [1] assumes that the underlying production possibility set (PPS) contains observed units and denoted by $T = \{(\boldsymbol{x}, \boldsymbol{y}) :\ \boldsymbol{x} \in \Re_+^m \text{ can produce } \boldsymbol{y} \in \Re_+^s\}$.

According to Banker et al. [1], the minimum extrapolation PPS which contained the observed units and free disposability and convexity assumptions can be explicitly stated as:

$$T_{\text{VRS}} = \left\{ (\boldsymbol{x}, \boldsymbol{y}) : \boldsymbol{x} \geqslant \sum_{j \in J} \boldsymbol{x}_j \lambda_j,\ \boldsymbol{y} \leqslant \sum_{j \in J} \boldsymbol{y}_j \lambda_j,\ \sum_{j \in J} \lambda_j = 1,\ \lambda_j \geqslant 0\ \forall j \in J \right\}$$

Note that if we assume all data are non-negative then we have $T_{\text{VRS}} \subseteq \Re_+^{m+s}$. Now, we assume that the production process yields a portion containing both positive and negative data. This could be occurring in both inputs and outputs stimulatingly, but here for simplicity of presentation, we assume that negative values appear only in some of the output variables. The extension of the obtained result to the general case could be straightforward.

Let partition the observed output vector $\boldsymbol{y}_j$ as $(\boldsymbol{y}_j^P, \boldsymbol{y}_j^N)$ where P is associated with the outputs in which for all $j \in J$ and $r = 1, \ldots, s$, $y_{rj} \geqslant 0$ and N is used to show the other outputs. Hence, the PPS with negative output data in variable returns to scale (VRS)[1] technology is stated as

$$T_{\text{VRS}} = \left\{ (\boldsymbol{x}, \boldsymbol{y}) :\ \boldsymbol{x} \geqslant \sum_{j \in J} \boldsymbol{x}_j \lambda_j,\ \boldsymbol{y} \leqslant \sum_{j \in J} \begin{pmatrix} \boldsymbol{y}_j^P \\ \boldsymbol{y}_j^N \end{pmatrix} \lambda_j, \right.$$
$$\left. \sum_{j \in J} \lambda_j = 1,\ \lambda_j \geqslant 0\ \forall j \in J \right\}$$

As it is shown in the literature the above formulation cannot be used directly to derive VRS DEA models when N is a nonempty set. To deal with the nonnegative values Emrouznejad et al. [6] replaced $\boldsymbol{y}_j^N$ by $\boldsymbol{y}_j^N = \boldsymbol{y}_j^1 - \boldsymbol{y}_j^2$ for all $j \in J$, where,

$$y_{rj}^1 = \begin{cases} y_{rj} & \text{if } y_{rj} \geqslant 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad y_{rj}^2 = \begin{cases} -y_{rj} & \text{if } y_{rj} < 0 \\ 0 & \text{otherwise} \end{cases}$$

According to the above notation, Emrouznejad et al. [6] defined the following PPS consisting of all $(\boldsymbol{x}, \boldsymbol{y}^P, \boldsymbol{y}^1, \boldsymbol{y}^2)$ satisfying in all of the constraints shown below.

$$\begin{aligned}
&\sum_{j \in J} \boldsymbol{x}_j \lambda_j \leqslant \boldsymbol{x} \\
&\sum_{j \in J} \boldsymbol{y}_j^P \lambda_j \geqslant \boldsymbol{y}^P \\
&\sum_{j \in J} \boldsymbol{y}_j^1 \lambda_j \geqslant \boldsymbol{y}^1 \\
&\sum_{j \in J} \boldsymbol{y}_j^2 \lambda_j \leqslant \boldsymbol{y}^2 \\
&\sum_{j \in J} \lambda_j = 1 \\
&\lambda_j \geqslant 0 \quad \forall j \in J
\end{aligned}$$

The above formulations show that PPS of SORM model [6] is similar to the PPS of VRS DEA model with non-negative

[1] It should be noted that the constant returns to scale (CRS) technology assumes that any DMU can be radially expanded or contracted to form other feasible DMU hence a CRS assumption for the technology is not consistent with the existence of negative data (see [13]).