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Linear calibration method of magnetic gradient tensor system

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ABSTRACT

Based on the detailed analysis of systematic errors, mathematical model of error parameters is constructed and linear calibration method is proposed for magnetic gradient tensor system. Firstly, nonlinear mathematical model of error parameters for single vector magnetometer is constructed based on scalar calibration, and least square solution is deduced by two nonlinear conversions without any mathematical simplification. Then outputs of four tri-axial magnetometers are calibrated to sensor's orthogonal coordinate respectively. Secondly, a least square estimation is proposed for the misalignment errors between different magnetometers according to the rotation matrix comprising conversion of different orthogonal coordinate system. After calibration, outputs of tri-axial magnetometers are acquired along the ideally platform frame-orthogonal coordinate system and these enable calibration of magnetic gradient tensor system. Simulations and experiments show that the proposed linear calibration method can accurately solve the detailed error parameters and decrease measurement errors of magnetic gradient tensor system remarkably.

1. Introduction

Magnetic gradient tensor measurements have many theoretical advantages over conventional magnetic surveys. For example, gradients of the magnetic field components are insensitive to orientation and rotation noise, which can offer better spatial resolution than magnetic field vectors and total magnetic intensity. In addition, gradient measurements have a high degree of immunity from regional background fields and diurnal variations [1–3]. In the last decades, magnetic measurement theories and sensor techniques have been widely improved. Several magnetic gradient tensor systems comprising fluxgate magnetometers or superconducting quantum interference devices have been developed, and some research institutes have done some tentative experiments of magnetic gradient tensor measurement [4–7].

Magnetic gradient tensor systems with different structural forms comprise multiple vector magnetometers. Because of the technological and material limitations in the magnetometer manufacturing process, there are some systematic sources of error, including hard and soft iron errors, sensor biases, scale factors and non-orthogonality, which seriously affect the accuracy of magnetometer. Magnetic gradient tensor errors may have thousands of nanoteslas and they have to be corrected. In addition, during the installation process of different magnetometers, displacement and rotation errors may be caused by the magnetometer move away and rotate according to the installation central point. The displacement errors caused by the mechanical technology could be neglected or obviated by higher precision machinery. However the rotation errors cannot be neglected and they bring misalignment errors, which must be calibrated carefully, between the different sensitive axes of magnetometers.

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There are some literatures about the magnetic sensor array calibration [8,9]. Without loss of generality, calibration methods of vector magnetometer could be passed onto the magnetic gradient tensor system comprising multiple magnetometers. Calibration approaches for vector magnetometer could be divided into vector calibration and scalar calibration method [10,11]. Vector calibration technique needs a 3D Helmholtz coil system to generate arbitrary magnetic field vectors, and then the sensor biases, scale factors, non-orthogonality angles could be obtained simultaneously. However a dedicated facility with coils, ultra-high precision tri-axial non-magnetic platform and other expensive equipment are required and it is impractical for in-field use. Sometimes the cost of vector calibration approach far outweighs cost of the magnetometer itself [12]. Scalar calibration, described as a "poor-man's" calibration method [13], does not require a high degree of expertise or an expensive hardware, and only needs a homogenous magnetic field and a scalar proton magnetometer which is used to measure the intensity of the magnetic field. Thus we consider scalar calibration for magnetic gradient tensor system.

Scalar calibration for magnetic gradient tensor system is mostly two-step method [14,15]. Calibration for the single vector magnetometer errors is provided as a first step and the second step is to calibrate the misalignment errors between different magnetometers. Yu et al. [16] achieved calibration of magnetic gradiometer, but second or higher order small quantities are neglected in the process of single magnetometer calibration model and the calibration deviations are brought in. Pang [17] used the Levenberg–Marquardt algorithm to achieve calibration of single vector magnetometer and calibrated the misalignment errors considering the first magnetometer as a reference. However, little work has been done on the calibration of magnetic gradient tensor system with linear method. On the other hand, choosing one of the magnetometers as reference magnetometer to correct the vector alignment differences for the other magnetometers cannot transform outputs of the magnetic gradient tensor system along the platform frame-orthogonal coordinate.

By classifying the systematic sources of error, nonlinear mathematical model for single vector magnetometer is constructed based on scalar calibration in this paper. Without any mathematical simplification, the nonlinear model is transformed to the linear model by two nonlinear substitutions and error parameters are deduced according to the least square method. Based on this, a least square estimation is proposed according to the rotation matrix comprising conversion of different orthogonal coordinate for the misalignment errors. So outputs of the magnetic gradient tensor system are converted along the platform frame-orthogonal coordinate. Simulations and experiments based on fluxgate magnetometers are tested and linear calibration of magnetic gradient tensor system is achieved.

2. Magnetic gradient tensor measurement principle and system

Magnetic gradient tensor is the spatial rate of change of the magnetic field vector in three orthogonal directions. If B denotes magnetic field vector and magnetic gradient tensor G can be written as multiplication of two matrices which contain three vector elements respectively.

$$G = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{bmatrix} [Bx \;\; By \;\; Bz] = \begin{bmatrix} B_{xx} & B_{xy} & B_{xz} \\ B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 U}{\partial x^2} & \frac{\partial^2 U}{\partial x \partial y} & \frac{\partial^2 U}{\partial x \partial z} \\ \frac{\partial^2 U}{\partial y \partial x} & \frac{\partial^2 U}{\partial y^2} & \frac{\partial^2 U}{\partial y \partial z} \\ \frac{\partial^2 U}{\partial z \partial x} & \frac{\partial^2 U}{\partial z \partial y} & \frac{\partial^2 U}{\partial z^2} \end{bmatrix} \tag{1}$$

where U denotes magnetic scalar potential, Bx, By and Bz are measured magnetic field components in three orthogonal directions, B_{pq}, $p, q = x, y, z$ denote magnetic gradient tensor component in different directions.

The geomagnetic field and magnetic anomaly caused by the ferromagnetic matter are magnetostatic fields which do not contain conduction currents. So the curl and divergence of magnetic field are equal to zero according to Maxwell's magnetostatic equations.

$$\begin{cases} \nabla \cdot B = \frac{\partial Bx}{\partial x} + \frac{\partial By}{\partial y} + \frac{\partial Bz}{\partial z} = 0 \\ \nabla \times B = 0 \end{cases} \tag{2}$$

According to (1) and (2), G is symmetric and traceless, only five of nine tensor components are independent.

It is not available to intrinsically measure second derivative of magnetic scalar potential and gradients of magnetic field vector. So measurements must estimate gradients by differencing vector measurements over short baseline [4]. However this approximation neglects high order derivatives of Taylor series. Herein, we define the neglected high order derivatives which arose by geometric configuration structure of the vector magnetometer array as structure errors, and the structure errors also affect accuracy of the magnetic gradient tensor measurements.

Several different configurations of magnetic gradient tensor system are analyzed in [18], simulation results denote that the plane cross tensor structure is the best and it has the minimal structural errors. Based on this, a cross magnetic gradient tensor system comprising four tri-axial magnetometers is designed in this paper and its sketch map is shown in Fig. 1. The x and y axes lie along the orthogonal baselines and the z axis is chosen to make a right-handed Cartesian coordinate system. Baseline distance between two magnetometers in the same direction is $2d$.

Tensor formula of the cross magnetic gradient tensor in the point O is shown as:

$$G = \begin{pmatrix} \frac{B_{1x}-B_{3x}}{2d} & \frac{B_{2x}-B_{4x}}{2d} & \frac{B_{1z}-B_{3z}}{2d} \\ \frac{B_{1y}-B_{3y}}{2d} & \frac{B_{2y}-B_{4y}}{2d} & \frac{B_{2z}-B_{4z}}{2d} \\ \frac{B_{1z}-B_{3z}}{2d} & \frac{B_{2z}-B_{4z}}{2d} & -\frac{B_{1x}-B_{3x}}{2d} - \frac{B_{2y}-B_{4y}}{2d} \end{pmatrix} \tag{3}$$

where B_{ij}, $i = 1, 2, 3, 4$, $j = x, y, z$ denote magnetic vector component in the j direction of the i-th magnetometer.

Structure errors of the cross magnetic gradient tensor system can be shown as follows [19].

$$\Delta\tilde{G} = \begin{Bmatrix} \frac{1}{3!}\frac{\partial^3 Bx}{\partial x^3}\Big|_o \cdot d^2 & \frac{1}{3!}\frac{\partial^3 Bx}{\partial y^3}\Big|_o \cdot d^2 & \frac{1}{3!}\frac{\partial^3 Bz}{\partial x^3}\Big|_o \cdot d^2 \\ \frac{1}{3!}\frac{\partial^3 By}{\partial x^3}\Big|_o \cdot d^2 & \frac{1}{3!}\frac{\partial^3 By}{\partial y^3}\Big|_o \cdot d^2 & \frac{1}{3!}\frac{\partial^3 Bz}{\partial y^3}\Big|_o \cdot d^2 \\ \frac{1}{3!}\frac{\partial^3 Bz}{\partial x^3}\Big|_o \cdot d^2 & \frac{1}{3!}\frac{\partial^3 Bz}{\partial y^3}\Big|_o \cdot d^2 & \frac{-1}{3!}\left(\frac{\partial^3 Bx}{\partial x^3}\Big|_o + \frac{\partial^3 By}{\partial y^3}\Big|_o\right) \cdot d^2 \end{Bmatrix} \tag{4}$$