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Analysis of the sensor placement for optimal temperature distribution reconstruction

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ABSTRACT

The temperature distribution as a function of time and space is reconstructed over a non-homogeneous media having an arbitrary three-dimensional geometry. This is done by applying an inverse problem to the collected data from optimally placed sensors on the boundary surface of the object under study. Sensor positioning and the choice of the number of sensors are optimized in terms of the resolution of the reconstructed temperature field and the error propagation of the method in case of uncertain measurements. The method can be performed in real time since the major computation burden is performed off-line.

1. Introduction

The operation and control of distributed systems usually requires precise information on the spatial dynamic variables of interest as a function of time. Often due to budget considerations one is not able to dedicate as many sensors as desired to achieve the required information. The reduction of sensor numbers requires introducing the model of the process as prior information. Typically in complex systems there is no analytical solution to this kind of models. In addition geometry and the physics of the problem may imply certain constraints for positioning the sensors. Hence, a systematic approach capable of enhancing the optimized solution for field reconstruction in terms of spatial resolution and uncertainty is quite crucial. The class of system considered here is the scalar field which can be described by partial differential equations (PDE). This paper mainly focuses on temperature field; however, the presented method can be expanded to the whole class of problems mentioned earlier. The work presented here, is aimed to analyze how the position and number of sensors can be optimally selected to enhance the best possible temperature distribution reconstruction in the existence of the physical constraints of the problem. Meanwhile it has been tried to avoid the computationally expensive analytical approaches to represent the system model (see [1] for instance) and to introduce a general practical on-line method that can be performed in real situation. Unfortunately the methods presented in literature mostly become limited to number of dimensions and complexity of the geometry of the object under study, since typically in such cases the process model gets too complicated. Consequently the major part of the available work has been dedicated to situations where the geometry of interest either owns less than three dimensions [1–7] or is a well shaped three dimensional geometry where the analytic solution is known [8,9].

The direct analytical approaches mainly fall in the two general categories, namely separation of variables and Integral Transform Technique (ITT) [10]. Although the latter one has more capabilities such as handling non-homogenous boundary conditions, still both of the methods often require solving complicated eigenvalue problems [8]. This

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issue has been treated in a more practical and efficient way in [11,12] implementing the so called Generalized Integral Transform Technique (GITT) [13]. The idea is to expand the unknown eigenfunctions of the mentioned eigenvalue problem by a chosen set of known basis via proposing a simpler auxiliary eigenvalue problem, transforming the PDE to denumerable ordinary differential equations (ODE). Numerical techniques will be then applied to achieve the solution of this system of equations. Finally the solution as in ITT should be transformed back to the original problem using inverse transform. This hybrid numerical–analytical approach is promising since it is also able to deal with diffusion–convection problems where the transportation of the mass is involved [14,15]. However, extra care should be taken regarding the introduction of auxiliary problem and transforming the boundary conditions.

While typically direct approaches are either impractical or complicated to apply, various inverse methods have been developed in temperature field reconstruction area. Sadly, the effort to find a general solution related to arbitrary complex heat evolution domains in the time of facing limitations over the availability of measurement sub domains and maximum number of sensors has been less fertile. The works concerning thermal map reconstruction via optimized sensor placement can be found in more specific areas such as microprocessors [16,17] although, in both inaccuracy of data and noisy readings from sensors have not been taken into account. Some efforts have been also established related to general field reconstruction from optimal sensor allocation [18,19]. Notably, an interesting work regarding this issue has been developed in [20] for general field reconstruction relied on a reduced model for distributed system representation along with an optimal sensor placement algorithm. However, the effect of noise on the stability of the algorithm in case of noisy measurements has not been explored. In [21] an exhaustive search has been carried out for optimal sensor positioning which becomes less efficient when the number of possible sensor location candidates grows.

A similar concept to this paper was recently introduced in [22] where the optimization of positioning the sensors was merely done by maximizing the resolution of the reconstructed field. While this approach may be true for some cases, generally one should take into consideration the key role of error propagation in inverse problems, meaning in the case of noisy measurements the final reconstruction may become unstable. This issue will be further explained in Section 3.1. Moreover, unlike [22] where the number of sensors to be used was mostly controlled by predefined constraints of the problem, here this number is suggested by the algorithm based on the conditions and type of the problem and may even be less than the maximum allowed number of sensors specified as a priori (see case study Section 5.2).

In the proposed approach the unknown temperature distribution is interpolated, as a function of time over a desired domain Γ (Reconstruction sub-domain), using just a few measurements placed in a sub domain Π (Observation sub-domain) of the object of interest, modeled in the volume Ω (Process volume), as shown in Fig. 1. To achieve

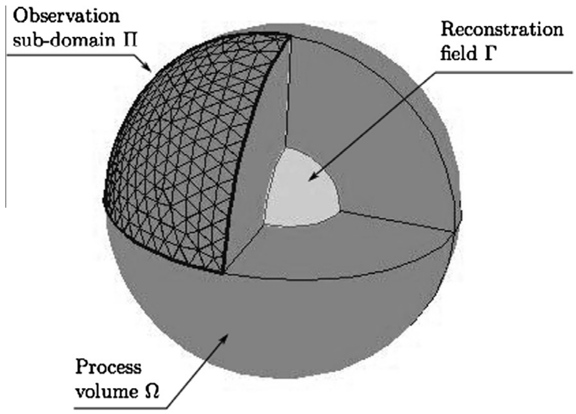

Fig. 1. Solution space.

this goal data assimilation [23] is implemented with the aim of fusing the observed data with the process model. The model is extracted from the PDE describing the heat evolution equations. In principle, the well known solutions to such equation problem can be categorized in two main, analytical and numerical groups. As mentioned earlier the existing pure analytical approaches lose their efficiency as the complexity of the problem increases – such as complicated boundary conditions, complexity of geometry or variety of consisting materials – for the sake of generality and practicality of the algorithm the numerical approach has been preferred here. It is worth noting that here the bulk of computation burden is done off line and once for all, providing the necessary ground for the algorithm to be applied on line at a low cost.

Here, the heat equation has been taken into consideration as Initial Boundary Value Problems (IVBP). Separation of time and space has led to possibility of capturing the space domain solution via solving a Sturm–Liouville eigenvalue problem while the time evolution information is encapsulated in the time dependent weight coefficients. The summation of properly weighted eigenfunctions will result to reconstruction of temperature map at any desired sub domain as a function of time. The solution to such an eigenvalue problem has been achieved numerically by implementing a Finite Element Method (FEM) software, namely Comsol Multiphysics. A novel technique has been applied in order to reduce the size of the system representation spanned by these eigenfunctions bases via taking into account the most relevant bases. Meanwhile evaluation of the weight coefficients is obtained from solving an inverse problem where the forward model is built based on the reduced solution space and observations are the sensed temperatures at the allowed observation sub domain Π. The implied optimization method is then responsible for revealing the optimized number and arrangement of sensors in order to enhance the reconstructed temperature field with minimum resolution error and minimum uncertainty. The available boundary measurement surfaces and maximum number of sensors to be used are inserted to optimization as constraints to be satisfied.