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Determination of optimum measurement points via A-optimality criterion for the calibration of measurement apparatus

Ch. Hajiyev *

Istanbul Technical University, Faculty of Aeronautics and Astronautics, Maslak, 34469 Istanbul, Turkey

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ABSTRACT

A procedure for optimal selection of the measurement points to get the best calibration characteristics (for the chosen optimality criterion) of measuring apparatus is proposed. The coefficients of the calibration characteristics are evaluated by the classical least squares method. For this work, the A-optimality criterion has been used as an optimality criteria. As an example, the problem of optimal selection of the standard pressure setters (the piston gauges) during calibration of the differential pressure gage is solved. Obtained values of the optimum measurement points for the calibration of the differential pressure gage are checked via actual experiments.

1. Introduction

Accurate measurement is the basis of almost all engineering applications, since uncertainty inherently exists in the nature of any measuring apparatus. On the other hand, the cost of a measuring apparatus increases with its accuracy. Therefore, low cost accurate measurement devices are one of the main goal of metrology engineers. One way for decreasing the sensor uncertainties is the calibration process [1]. Therefore, this paper deals with the calibration of a low cost sensor using the corresponding values obtained by reference standards. From the practical point of view, the calibration characteristics should be in a polynomial form. The accuracy of this polynomial depends on the noise-free data which is used to obtain the characteristics [2]. To reduce the effect of the noise, excessive number of data should be used. However this requires more experiments and that increases the cost. Thus, the main question becomes the evaluation of accurate calibration characteristics with minimum number of experimental data. In [1–4] the equidistant measurement points are used for calibration of measurement apparatus. On the other hand, even though it is paradoxical, the application of the equidistant measurement points to get the best calibration characteristic (for the chosen optimality criterion) is erroneous, because these points are not optimum in a sense that given performance criterion is minimum. The equidistant calibration points are obtained by simply dividing the range of the transducer into the appropriate number of samples.

In [5,6] an approach to design sensor calibration is proposed with the aim of reducing the calibration curve uncertainty. This uncertainty reduction is achieved by minimizing either the standard deviations of the regression curve coefficients or the standard deviation of the whole estimated calibration curve. In particular, criteria for the choice of the number of calibration points and their optimum location are theoretically identified when the response characteristic is a polynomial and the uncertainties of the sensor outputs can be neglected. The proposed criteria is not directly applicable to non-linear sensors and complex apparatus with an indirect estimation of the measurand.

The calibration of sensors, or in general, the measurement apparatus, can be considered as a typical problem of "model searching" from experimental data and then, it should be approached by applying experimental design

* Tel.: +90 2122853105; fax: +90 2122852926.
E-mail address: cingiz@itu.edu.tr

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techniques. In this case, the calibration design should drive the operator in the choice of [7]:

- the experimental plan (e.g. the number and the location of the calibration points, the number of the repetitions);
- the main influence quantities;
- the regression curve;
- the regression technique;
- the standard references and their uncertainties.

The calibration design using the experimental design techniques is proposed in [7], when the relationship between the indirectly calculated measurands and the sensor inputs is non-linear or more than one measurand has to be considered. In particular, the optimum calibration plan for measurement chain is identified by suitably elaborating the error propagation law suggested by the ISO Guide [8].

Sensor calibration and compensation using the artificial neural network are performed in [9]. The artificial neural network based inverse modeling technique is used for the sensor response linearization. The choice of the order of the model and the number of the calibration points are important design parameters in this technique. An intensive study about the effect of the order of the model and the number of the calibration points on the lowest asymptotic root-mean-square (RMS) error has been reported in this paper.

In [10] a genetic algorithm is used to perform optimizations for both the computation of the optimal input to the sensor and the optimal constant feedback gain.

A calibration method, which uses the neural network and genetic algorithms together, is presented in [11]. It uses the improved back-propagation neural network to model the characteristic curve of the vortex flowmeter. And then it applies the genetic algorithms to seek two additional optimum calibration points intelligently at the intervals where the curve are non-linear obviously. At last the vortex flowmeter is calibrated at the new calibration points. The methods based on the artificial neural networks and genetic algorithms do not have physical bases. Therefore according to the different data, which corresponds to the same event, the model gives different solutions. Thus, the model should be continuously trained by using the new data.

In this study, optimal selection method of the sample measurement points via A-optimality criterion for the calibration of measurement apparatus, which takes the uncertainties on their outputs into account, is proposed.

2. Problem statement

As it is known from practical considerations, the calibration curve should be in a polynomial form as follows:

$$y_i = a_0 + a_1 p_i + a_2 p_i^2 + \cdots + a_m p_i^m, \tag{1}$$

where y_i is the output of the low cost transducer, p_i are the values of the reference standard and $a_0, a_1, \ldots, a_m$ are the calibration curve coefficients. Measurement contains random noises in Gaussian form

$$z_i = y_i + \delta_i = a_0 + a_1 p_i + a_2 p_i^2 + \cdots + a_m p_i^m + \delta_i \tag{2}$$

where z_i is the measurement result, δ_i is measurement error with zero mean and standard uncertainty σ. Let the calibration curve coefficients be denoted as $\tilde{\theta} = [a_0, a_1, \ldots, a_m]^T$.

The coefficients in these polynomials are evaluated in [12] by the least squares method. The expressions that are used to make the evaluation had the form:

$$\begin{aligned} \tilde{\theta} &= (\widetilde{X}^T \widetilde{X})^{-1} (\widetilde{X}^T z) \\ \widetilde{D}(\hat{\theta}) &= (\widetilde{X}^T \widetilde{X})^{-1} \sigma^2 \end{aligned} \tag{3}$$

where $z = [z_1, z_2, \ldots, z_n]^T$ is the vector of the measurements;

$$\widetilde{X} = \begin{bmatrix} 1 & p_1 & p_1^2 & \cdot & \cdot & \cdot & p_1^m \\ 1 & p_2 & p_2^2 & \cdot & \cdot & \cdot & p_2^m \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & p_n & p_n^2 & \cdot & \cdot & \cdot & p_n^m \end{bmatrix} \tag{4}$$

is the matrix of the known coordinates (here, $p_1, p_2, \ldots, p_n$ are the values that are producible by the reference standard instrument), $\widetilde{D}(\hat{\theta})$ is the dispersion matrix of the estimated coefficients.

The values of the reference standard $p_1, p_2, \ldots, p_n$, should be such that the polynomial characteristics given by (1) approximates the real calibration characteristics in the best form. Thus the problem can be stated as follows: Find the values of $p_1, p_2, \ldots, p_n$ such that the values of $a_1, a_2, \ldots, a_m$ are optimum in the sense that a given performance criterion is minimum.

As mentioned above, the matrix $\widetilde{D}$ can be used as a measure of the error between the low cost transducer and the high precision reference standard instrument. A performance criteria for the minimum of the matrix $\widetilde{D}$ can be selected by the following ways:

- Minimize the trace (the sum of the diagonal elements) of the matrix $\widetilde{D}$ (the A-optimality criterion).
- Minimize the generalize dispersion (the determinant) of the matrix $\widetilde{D}$ (the D-optimality criterion).
- Minimize the maximal eigenvalue of the matrix $\widetilde{D}$ (the E-optimality criterion).
- Minimize the sum of the all elements of the matrix $\widetilde{D}$ and etc.

Each of the above-mentioned measures characterizes one or another geometrical parameter of the correlation ellipsoid. The choice of one or another scalar measure as the optimality criteria for a solved particular problem depends on the basic indicators of optimization (the mathematical simplicity, convenience of obtaining analytical results, volume of computational expenditures, etc.). For this reason, A-optimality criterion is used in this work, i.e.

$$\min_{P_i} [Tr(\tilde{X}^T \tilde{X})^{-1} \sigma^2] \tag{5}$$

is sought. The values of $p_1, p_2, \ldots, p_n$, found by solving the above equations, should be in the range of 0–$p_{\max}$. Otherwise the solution is invalid.