Measurement 50 (2014) 244–249

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

On the input/output reduction in efficiency measurement

A. Hadi-Vencheh [a], Z. Ghelej Beigi [b,*], K. Gholami [c]

[a] Department of Mathematics, Khorasgan Branch, Islamic Azad University, Khorasgan, Isfahan, Iran
[b] Department of Mathematics, Mobarakeh Branch, Islamic Azad University, Mobarakeh, Isfahan, Iran
[c] Department of Science, Bushehr Branch, Islamic Azad University, Bushehr, Iran

ARTICLE INFO

Article history:
Received 20 May 2013
Received in revised form 10 December 2013
Accepted 12 December 2013
Available online 25 December 2013

Keywords:
Data envelopment analysis
Inputs/outputs reduction
Efficiency

ABSTRACT

Inputs and outputs reduction is an important discussion in management science. In some cases decision makers (DMs) forced to reduce some inputs and as a result they have to reduce some outputs. In this paper we propose a new method to find how much some inputs/outputs of each decision making unit (DMU) should be reduced such that the total efficiency of all DMUs after reduction being maximized. For this purpose we propose an improved DEA-like model. The proposed model decreases the total inefficiency of all DMUs. We employ a set of real data to show applicability of the proposed method and the results are compared with those find in the literature.

1. Introduction

Due to the limitation of resources, inputs and outputs reduction is a significant issue in the field of management. In real life, managers forced to reduce the resources like prohibition and bankruptcy. Many factories do not have enough raw materials therefore they have to reduce the resources and as a result the outputs should be reduced as well. Thus, inputs/outputs reduction is an important argument to governments, researchers and business managers. Data envelopment analysis (DEA) is anon-parametric method for measuring the relative efficiency of DMUs [1]. So far, numerous application of DEA has been reported in the literature [2]. One of the most important of applications of DEA is allocation, i.e., resource allocation, cost allocation and target setting. First we clarify the difference between terms of "resource allocation", "cost allocation" and "target setting" in the DEA, following Beasley [3]. The resource allocation may happen when the organization has restricted input resources or restricted output possibilities. In such circumstances, the organization must allocate the fixed input/output levels optimally among the DMUs. The cost allocation may happen when the organization has overheads/fixed costs that it has to allocate the fixed input levels optimally among the DMUs. The target setting may happen when the organization has no restrictions. The resource reduction is related to limitation when organization must reduce the inputs. Cook and Kress [4] were the first who propose a method to allocate fixed cost to DMUs fairly, but their method cannot allocate cost directly. Jahanshahloo et al. [5] proposed a simple formula to allocate fixed cost fairly, their method does not need solving any linear programming. Cook and Zhu [6] proposed a model to develop the method in [4], their method allocates cost among DMUs fairly, but Lin [7] showed that their method is infeasible and proposed a method to allocate fixed cost. Li et al. [8] proposed a model to allocate fixed cost where the fixed cost as a complement of other cost inputs. Hosseinzadeh Lotfi et al. [9] proposed an alternative method for resource allocation and target setting. They proposed an allocation mechanism that is based on a common dual weights approach. Wu et al. [10] and Li et al. [11] extended the previous studies by considering both the desirable and undesirable outputs in resource allocation. Wu et al. [10] proposed some DEA models for resource allocation which minimizes the undesirable outputs and maximize desirable outputs. Li et al. [10] proposed a multi objective linear

* Corresponding author. Tel.: +98 9178084606.
E-mail addresses: ahadi@khuisf.ac.ir (A. Hadi-Vencheh), zahra.ghlichbegi@yahoo.com (Z. Ghelej Beigi).

0263-2241/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.measurement.2013.12.016

programming for resource allocation and considered reduction for desirable inputs and undesirable outputs. For more studies see Golany and Tamir [12], Bi et al. [13], Korhonen and Syrjänen [14].

As the foregoing survey shows, most papers study resource allocation, target setting and allocate fixed cost. However, in most real life situation we concern with organizations with several branches and limited budgets. Hence, reducing the inputs yields to decreasing the outputs. To be noted that proposing a method for inputs/outputs reduction in DEA literature is rare. A search for this subject returned the recent papers by Amirteimoori and Emrouznejad [15,16]. In [15] the authors proposed a method for input/output reduction such that efficiency of each DMU after data reduction is better than its previous value. Besides, Amirteimoori and Emrouznejad [16] proposed a method for input/output reduction such that the total efficiency of the DMU after data reduction improves or does not change.

In this paper we propose a method for input/output reduction, where our aim is to get the reduced value of inputs/outputs such that the inefficiency of all DMUs is minimized. In the proposed model it is not important after reduction the efficiency of some DMUs be less than their efficiency before data reduction. In fact, our goal is to maximize the total efficiency of the DMU after reduction.

This paper organized as follow. In the following section we review the conventional DEA models. Section 3 is devoted to proposing our method for input/output reduction. In Section 4 we compare our method with those in the literature. Section 5 includes a numerical example as well as an application of proposed model. Finally, the conclusions are given in Section 6.

2. Preliminaries

Suppose that we have n independent DMUs (DMU_j $(j = 1,\ldots,n)$) such that each DMU by using m inputs x_{ij} $(i = 1,\ldots,m)$ produce s outputs $y_{rj}(r = 1,\ldots,s)$. Let DMU_o, $o \in \{1,\ldots,n\}$, be under consideration. The following model has been proposed for evaluation the efficiency of DMU_O [2]:

$$\begin{aligned} &Max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ &s.t. \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad j = 1,\ldots,n, \\ &\quad u_r \geqslant \varepsilon, \qquad r = 1,\ldots,s, \\ &\quad v_i \geqslant \varepsilon, \qquad i = 1,\ldots,m. \end{aligned} \tag{1}$$

where ε is non-Archimedean constant, u_r and v_i are weights corresponding to rth outputs and ith inputs respectively. Model (1) is a fractional programming model and it is easy to see that it can be converted to the following input-oriented DEA model:

$$\begin{aligned} e_o = Max \quad & \sum_{r=1}^{s} u_r y_{ro} \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{io} = 1, && i = 1,\ldots,m, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leqslant 0, && j = 1,\ldots,n, \\ & u_r \geqslant \varepsilon, && r = 1,\ldots,s, \\ & v_i \geqslant \varepsilon, && i = 1,\ldots,m. \end{aligned} \tag{2}$$

3. Inputs and outputs reduction

Following Amirteimoori and Emrouznejad [16] suppose I is the set of indices to the inputs that should be reduced and $I' = \{i_1,\ldots,i_m\} - I$. Also O be the set of indices corresponding to the outputs that should be reduced and $O' = \{O_1,\ldots,O_s\} - O$. Suppose that the total reduction in ith input is $C_i (i \in I)$ and the total reduction in rth output is $F_r (r \in O)$. In this paper we answer the following question: how much inputs/outputs of each DMU should be reduced such that the total efficiency of all DMUs is being maximized. To answer this question consider the following system:

$$\begin{aligned} & \frac{\sum_{r\in O'} u_r y_{rj} + \sum_{r\in O} u_r (y_{rj} - f_{rj})}{\sum_{i\in I'} v_i x_{ij} + \sum_{r\in I} v_i (x_{ij} - c_{ij})} = 1, && j = 1,\ldots,n, && \text{(i)} \\ & \sum_{j=1}^{n} c_{ij} = C_i, && i \in I && \text{(ii)} \\ & \sum_{j=1}^{n} f_{rj} = F_r, && r \in O && \text{(iii)} \\ & c_{ij} \leqslant x_{ij}, && i \in I,\ j = 1,\ldots,n, && \text{(iv)} \\ & f_{rj} \leqslant y_{rj}, && r \in O,\ j = 1,\ldots,n, && \text{(v)} \\ & c_{ij} \geqslant 0, && i \in I,\ j = 1,\ldots,n, \\ & f_{rj} \geqslant 0, && r \in O,\ j = 1,\ldots,n, \\ & u_r \geqslant \varepsilon, && r = 1,\ldots,s, \\ & v_i \geqslant \varepsilon, && i = 1,\ldots,m. \end{aligned} \tag{3}$$

Table 1
The data of Example 1 and the value of input and output reduction.

DMU_j	x_1	x_2	y_1	y_2	c_{1j}	f_{2j}	e_j	e_j^{new}	e_j^{newAE}
1	185	235	57	59	42.47911	0	0.5855	1	0.6198
2	173	224	65	91	0	24.66537	0.9384	1	0.8123
3	159	145	79	84	0	30.45952	1	1	1
4	158	199	83	88	0	30.13762	1	1	1
5	175	221	74	73	0	7.162211	0.8031	1	0.8450
6	159	212	85	69	0	9.817863	1	1	1
7	199	201	48	70	11.86718	0	0.6416	1	0.6740
8	177	189	64	99	0	34.60394	1	1	1
9	176	149	59	74	0	13.15755	0.8552	1	1
10	191	191	81	61	20.63271	0	0.8381	1	0.9758
Sum					74.979	150.0041	8.6619	10	9.2604