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Modeling and analysis of an over-constrained flexure-based compliant mechanism

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ABSTRACT

Bridge-type micro-displacement amplifier with flexure hinges is a classic displacement amplification mechanism. Existing theoretic models cannot predict its amplification ratio and input stiffness accurately and make it very difficult to confirm the amplifier's performance and error compensation by means of these models, which is very significant in ultra-precision positioning. This paper focuses on the development of design equations that can accurately calculate the ideal displacement amplification ratio and input stiffness of the amplifier based on the thought of statically indeterminate structure. Force Method, Maxwell–Mohr Method, principle of superposition and deformation compatibility are used together to establish uncanonical linear homogeneous equations. The analytical results are verified by FEA simulations. The influence of the geometric parameters on the amplifier performance is investigated. It is noted that amplifier performance is more sensitive to the longitudinal distance of flexure hinges. Besides, two same-sized amplifiers with the opposite output directions can be clearly differentiated by these equations.

1. Introduction

In recent years, piezoceramics (PZTs) are employed in many industrial processes due to their high positioning accuracy and resolution, compact profile, no-need of lubrication, etc. On the one hand, on account of their piezoelectric effect, piezoelectric ceramic filters, sensors and resonators are widely used in signal processing [1–4]. On the other hand, piezoelectric actuators are used in ultra precision positioning, e.g. atomic force microscopes (AFM), scanning probe microscope (SPM), laser-based confocal microscope, by utilizing their inverse piezoelectric effect [5–7]. For a small output motion range, a piezoelectric actuator can be directly used to achieve the output displacements. However, a large output motion range is required in many applications. Considering the travel stroke of a stacked piezoelectric actuator is relative to the length of the PZT, a displacement amplification mechanism is an effective method to amplify the stroke of PZT [8].

In the literature, many studies have been conducted to design and investigate several types of compliant mechanisms. Lobontiu and Garcia [9] analyzed a classic bridge-type compliant mechanism and presented a mathematical model for amplification ratio and input/output stiffness calculations of this stage based on Castigliano's second theorem and the strain energy. Another approach was followed by Ma et al. [10] in developing and proposing the displacement by using kinematic theory and elastic beam theory. The relationship between the ratio and design parameter has also been analyzed. Li and Xu [11] designed a compound bridge-type amplifier and discussed several previously analytical models for this compliant stage. After that, they developed closed form equations to predict the static properties, based on the Euler–Bernoulli beam theory. But the displacement amplification ratio is nonlinear compared with that of FEA. Koseki et al. [12] put forth

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the modeling method derived from the matrix method. This method is applicable to general flexure-based compliant mechanisms. Followed this approach, Li [13–15] and Ye [16] applied the matrix method to the bridge-type micro-positioning stages respectively. Similarly, Kim [17] established the stiffness matrix of a three-dimensional bridge-type amplifier mechanism by using matrix method. Yang et al. [18,19] designed the characterization of a single-axis, low-profile micro-positioning mechanism by using a multi-lever structure and proposed equations to predict the stiffness and displacement of a general lever configuration. Mottard [20] focused on the flexure arrangement scheme and developed a simplified analytical model to increase stage performance. Han et al. [21] designed a two-step amplification mechanism and gave Lagrange's equation to calculate amplification ratio and optimize its performance.

Based on the summary of the previous works, one can find that most of them have only taken the characteristic of compliant mechanism's geometric symmetry into account; therefore, the amplifier 1/4 geometric model is regarded as the research object in their works. Different from before, this work not only takes the symmetry of the amplifier into account but also considers the conditions of applied force and the amplifier constrains. Therefore 1/2 geometric model is taken as the analytical object. The main purpose of this paper is to introduce conception of statically indeterminate structure in the compliance mechanism and to develop general linear homogeneous equations for the lever-type or bridge-type amplifier which can be easy to implement numerically.

The remainder of this paper is structured as follows: analytical static modeling by using uncanonical Force Method [22] is performed in Section 2. Section 3 verifies the analytical predictions with the finite element analysis simulations. Then, the influences of geometric design parameters on the amplifier performance are discussed in Section 4 and conclusions are summarized in Section 5.

2. Modeling

2.1. Derivation of analytical model

A schematic diagram of the ordinary bridge-type amplifier is illustrated in Fig. 1 with its axis of symmetry, global coordinate system whose origin is on the amplifier's centroid, direction of the output displacement and applied forces. As can be observed that, on the one hand, the mechanism employs flexure hinges at all joints and levers are symmetric about x axis and y axis, respectively; on the other hand, the external loadings produced by PZT and the fixed support are only symmetric about x axis. According to the rules of simplified calculation about symmetric statically indeterminate structure, 1/2 model is separated from original. For the purpose of being equivalent with prototype in 1/2 model, antisymmetric internal force is neglected. The equivalent schematic diagram is depicted in Fig. 2 with its geometric dimensions.

Referring to Fig. 2, normal force X_1 and bending moment X_2 exert on the symmetric cross section. By inspection, the frame is statically indeterminate to the second degree and X_1 and X_2 will be considered as the redundants. Unlike traditional statically indeterminate structure, there are two additional unknown conditions with regard to the amplifier. First, input force F_{in} is no longer a given force, and second, the vertical displacement at L would be calculated. Therefore, the four compatibilities of displacement equations are needed. For the sake of unification, F_{in} is replaced by symbol X_3, and a dummy vertical unit load X_L acting on the frame only for calculating the vertical displacement at L.

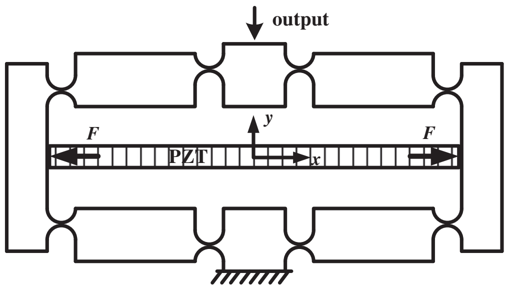

Fig. 1. Schematic of bridge-type amplifier with flexure hinges.

In Fig. 2, four equations can be established as follows. By principle of superposition, the displacement, produced by X_i (i = 1, 2, 3), sums up the total horizontal displacement at L. Meanwhile, according to deformation compatibility, the amount horizontal deformation Δ_1 is zero. The first compatibility equation is:

$$\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 = \Delta_1 = 0 \quad (1)$$

where δ_{1j} means the corresponding horizon displacement at L when X_j (j = 1, 2, 3) acting on the frame.

Similarly, another canonical compatibility equation can be established with regard to the rotation at L. By utilizing deformation coordination condition, the amount rotation Δ_2 along with X_2 is zero:

$$\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 = \Delta_2 = 0 \quad (2)$$

where δ_{2j} is the corresponding angular displacement with respect to X_j.

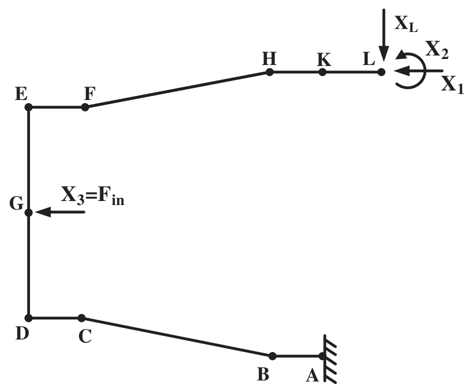

Fig. 2. Schematic of amplifier equivalent system.