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An alternative method for measuring small displacements with differential phase difference of dual-prism and heterodyne interferometry

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ABSTRACT

This study proposes an optical method for measuring small displacements. In this method, two semispherical prisms are used as test apparatuses. Because testing lights inside the prisms are at polarization angles, the phase difference can be differentially magnified twice and the measurement resolution can be greatly increased. Furthermore, using common-path heterodyne interferometery ensures that this method is simple to implement, high accurate, and high stability against the vibration of the surrounding environment. The capability of this method was demonstrated with a sensitivity of 42.92°/mm and a resolution of 23.29 nm.

1. Introduction

The measurements of small displacement have been widely applied in different fields. As nanotechnology is being developed rapidly, the dimensions of related production have been reduced to a scale of sub-micrometer or nanometer. Therefore, the question of how to develop an accurate electro-optical testing technique for the measurement of small displacements has become an important issue. Several electrooptical methods have been proposed for this purpose. In Benedetto's method [1], a laser is inclined to hit a moving surface. The reflected light is deflected by a convex lens and finally reaches a position-sensitive detector (PSD). With the triangular method, the relationship between the displacement and the position on the PSD can be obtained. Other methods [2,3] use a charge-coupled device (CCD) array or photodetectors to detect the diffracted light of a moving grating and base on the principle of the Doppler Effect for solving the displacements. Conversely, optical interferometry is another method that is applied frequently for determining small displacements [4–11]. Such methods employ CCD to record vibrations produced by interferometric signals, and yield the functionality of rapid and real-time measurement [4,5]. In addition, some methods combine Michelson interferometry with heterodyne interferometric techniques for the measurement of small displacements with high resolutions [6–9]. Recently, Chiu et al. based on the total internal reflection (TIR) and the surface plasmon resonance (SPR), proposed a common-path heterodyne interferometry for high-resolution measurements of small displacements [10,11].

This study proposes an alternative method for measuring small displacements with the differential phase difference of dual-prism and heterodyne interferometry. A circularly polarized heterodyne light source is reflected by the mirror and separated into two parts by the beam splitter. The lights are respectively incident into two semispherical prisms at the bases of the prisms with polarization

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angles. The reflected beams then pass through two properly oriented analyzers for interferences. The phase difference of the interference signals can be differentially magnified twice, and is significantly sensitive to the small displacement of the mirror. Based on these effects, the resulting phase difference makes it possible to analyze the displacement of the mirror by using a heterodyne interferometry technique. The feasibility of this method has been verified by an experiment, and the displacement measurement resolution is approximately 23.29 nm. The method of measurement has the merits of being non-contact, involving a common-path configuration, with ease of operation, and yielding high stability and high resolution.

2. Principles

2.1. Relationship between small displacement and incident angles at the bases of dual semispherical prisms

Fig. 1 shows the geometric relationships of the displacement of mirror and the incident angles. For convenience, counterclockwise and clockwise angles are respectively defined as positive and negative. In Fig. 1, light is incident on the mirror at an angle θ_i; the reflected light is separated by the beam splitter and are normally incident into two semispherical prisms. In these prisms, the lights are respectively incident at the bases of the prisms with angles of θ_{B1} and θ_{B2}, and are reflected again. When the mirror undergoes a small displacement Δd, the lights' are incident on the prisms at angles of θ_1 and θ_2, then refract into prism at angles of θ' and θ''. Accordingly, the relationship between the incident angles of θ_1 and θ_2, and the small displacement d can be written as

$$\theta_1 = -\sin^{-1}\left(\frac{2\Delta d \sin\theta_i}{R}\right) \tag{1a}$$

and

$$\theta_2 = \sin^{-1}\left(\frac{2\Delta d \sin\theta_i}{R}\right), \tag{1b}$$

where R is the radius of the semispherical prism. According to Snell's law, the relationship between the transmission

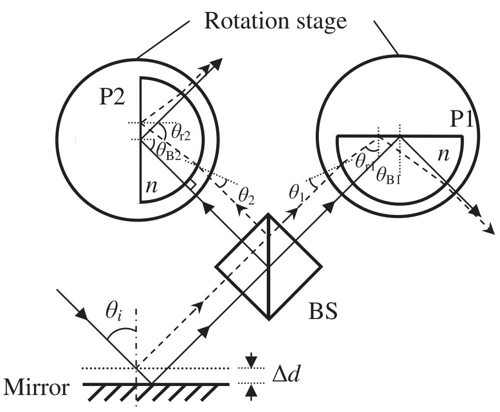

Fig. 1. Geometric relationships of the displacement of mirror and the incident angles.

angles of θ' and θ'', and the small displacement d are respectively given by

$$\theta' = -\sin^{-1}\left(\frac{2\Delta d \sin\theta_i}{n_1 R}\right) \tag{2a}$$

and

$$\theta'' = \sin^{-1}\left(\frac{2\Delta d \sin\theta_i}{n_1 R}\right), \tag{2b}$$

where n_1 represents the refractive index of the semispherical prism. Then, the refracted lights are respectively incident on the bases of the prisms at angles of θ_{r1} and θ_{r2}, and are again reflected. The changed incident angles θ_{r1} and θ_{r2} are given by

$$\theta_{r1} = \theta_{B1} - \theta_1 + \theta' \tag{3a}$$

and

$$\theta_{r2} = \theta_{B2} - \theta_2 + \theta'', \tag{3b}$$

respectively. Substituting Eqs. (1) and (2) into Eq. (3) enable the relationship between the changed incident angles, θ_{r1} and θ_{r2}, and the displacement Δd to be expressed as

$$\theta_{r1} = \theta_{B1} + \sin^{-1}\left(\frac{2\Delta d \sin\theta_i}{R}\right) - \sin^{-1}\left(\frac{2\Delta d \sin\theta_i}{n_1 R}\right) \tag{4a}$$

and

$$\theta_{r2} = \theta_{B2} - \sin^{-1}\left(\frac{2\Delta d \sin\theta_i}{R}\right) + \sin^{-1}\left(\frac{2\Delta d \sin\theta_i}{n_1 R}\right). \tag{4b}$$

From Eq. (4), the displacement Δd can be derived as

$$\Delta d = \sqrt{\frac{n_1^2 R^2 [1 - \cos((\theta_{r2} - \theta_{r1}) - (\theta_{B2} - \theta_{B1}))]}{8\sin^2\theta_i \left[1 + n_1^2 - 2n_1 \cos\frac{1}{2}((\theta_{r2} - \theta_{r1}) - (\theta_{B2} - \theta_{B1}))\right]}}. \tag{5}$$

Eq. (5) shows that the displacement Δd of the mirror is a function of incident angles θ_{r1} and θ_{r2}. When the refractive index n_1 and radius R of the semispherical prism and the original incident angles θ_{B1} and θ_{B2} are known, the displacement Δd can be obtained by an accurate measurement of θ_{r1} and θ_{r2}.

2.2. Measuring the variations of incident angles at the bases of dual semispherical prisms

Fig. 2 shows the optical setup for measuring the variations of incident angles at the bases of dual semispherical prisms. In other words, the +z-axis is set as the light propagation direction, and the x-axis is perpendicular to the paper plane. A horizontally polarized light passes through an electro-optic modulator (EOM) (with the fast axis at 45° relative to the x-axis), driven by a function generator with an angular frequency ω. The light passes through a quarter wave-plate (Q) with the fast axis aligned with the x-axis. Consequently, a circularly polarized heterodyne light source can be obtained [12,13]. The circularly polarized heterodyne light source is reflected by the mirror and separated into two parts by the beam splitter, which are respectively reflected again by the two semispherical prisms, P_1 and P_2. These beams pass through analyzers