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# Multisine excitation design to increase the efficiency of system identification analysis through undersampling and DFT optimization

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Multisine excitations are commonly employed to measure the frequency response of a nonlinear system. A typical pseudo-logarithmically-spaced frequency distribution may be oversampled at a rate significantly greater than the Nyquist rate to ensure accurate signal reconstruction without aliasing. In this paper, two algorithms are presented for optimizing the frequency distribution of a multisine excitation signal such that the system output can be undersampled without corruption, resulting in compact DFT bin utilization. In addition, the optimized excitation signal is designed to approximate a user-defined frequency distribution and make allowances for harmonic frequencies generated by system nonlinearities. Results show that at least an eleven-fold improvement in DFT bin utilization is possible for an example two-decade logarithmically-spaced 25-tone excitation signal applied to a nonlinear system exhibiting both second and third order harmonics. The use of optimized excitation signals in power constrained applications, such as structural health monitoring, can help to increase adoption rates by reducing system complexity and power source requirements.



## 1. Introduction

In system identification applications, it is common practice to apply a single sine or multisine input to a system under test and then measure the response. Typically, the output signal is digitally sampled at a greater-than-Nyquist rate to ensure that frequency aliasing does not corrupt signal measurements. Nonlinear systems can produce harmonics that demand even higher sampling rates. Existing analog and digital circuitry are available to implement these sampling and processing functions, but the cost in power and complexity can be substantial. For example, hardware implementations consume increasing power when operating at increasing frequency.

The primary objective of this paper is to present methods to design multisine input signals that, when applied to linear or nonlinear systems, can be sampled at less-than-Nyquist frequencies without signal loss or corruption. The benefits of undersampling are twofold. First, the measurement hardware operates at a lower frequency and thereby consumes less power. Second, signal design ensures controlled aliasing during undersampling that substantially improves DFT bin utilization and allows the signal to be processed in a more computationally-efficient manner. Furthermore, these signals are designed to intelligently accommodate harmonics produced by system nonlinearities. Taken together, the results are decreased power consumption and decreased processing complexity.

This paper is organized into six sections, beginning with this introduction. Additional background and significance are provided in Section 2. The requirements for designing a multisine excitation signal that can be undersampled, while still providing means for nonlinear detection through the measurement of select harmonics, are discussed in Section 3. These requirements present a discrete optimization problem with the goal of minimizing the total number of DFT bins needed to analyze an undersampled output signal with nonlinear detection. Using the frequency distribution **f**

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of the excitation signal as a free variable, the minimum number of DFT bins is directly calculated for small values of $M$ and for different sets of detection harmonics. In Section 4, the Min$N$-Free**f** algorithm is presented to identify frequency distributions that minimize the number of DFT bins required for larger values of $M$. These results set the lower bound for the number of required DFT bins for any frequency distribution **f**. Building on this, Section 5 presents the Min$N$-Target**p** algorithm that considers additional excitation signal requirements such as the acceptable error allowed between the resulting distribution designed for undersampling and the target excitation frequency distribution required by a particular system identification application. Finally, the capabilities of the Min$N$-Target**p** algorithm are investigated in Section 6 through its application to target logarithmic frequency distributions. Results show that considerable improvements in both sample frequency and DFT bin utilization are possible compared to Nyquist-sampled output signals that support nonlinear detection.

## 2. Background and significance

Widespread adoption of in situ structural health monitoring (SHM) to autonomously assess the condition and deterioration of real word infrastructure is curtailed by initial capital and recurring maintenance costs. Primary contributors to these prohibitors generally include the complexity of the instrumentation equipment and the need for localized power sources, which may require routine service for continued operation. SHM instrumentation requirements are satisfied with a commercial impedance analyzer, such as the HP4194A, but previous engineering efforts have led to simplified hardware setups. Self-contained DSP-based measurement modules utilizing off-the-shelf components can readily satisfy the demands of SHM and are orders of magnitude more economical compared to lab-grade equipment [1]. Likewise, complete instrumentation systems for SHM have been realized in single integrated circuits [2,3], further decreasing equipment costs and vastly reducing power consumption through integration. Additional system optimization has been realized by substituting classical, high-resolution measurement techniques with less intensive measurement approximations that are specifically designed with the goal of improving energy efficiency and reducing system complexity [4]. In combination, these approaches have opened the door for energy harvesting [5–7] as a viable, low maintenance alternative to conventional energy storage sources such as batteries.

The power consumption of digital circuitry in a standard CMOS process, a low cost process technology for integrated SHM sensors, can be divided into dynamic and leakage power. Dynamic power is modeled as

$$P_{\mathrm{D}} = \alpha C f V_{\mathrm{DD}}^2, \tag{1}$$

where $\alpha$ is the activity factor, $C$ is the total capacitance of the switching circuits, $f$ is the switching frequency, and $V_{\mathrm{DD}}$ is the supply voltage. Likewise, leakage power is modeled as

$$P_{\mathrm{LEAK}} = V_{\mathrm{DD}} I_{\mathrm{LEAK}}, \tag{2}$$

where $I_{\mathrm{LEAK}}$ is dominated by the drain-to-source current of the transistor when the gate-to-source voltage is zero [8]. The 0.18 μm process is a typical design node for low-power mixed-signal integrated circuits because it offers a good balance between analog capabilities, digital integration, IP availability, and manufacturing cost. In this geometry, dynamic power consumption is the predominant concern for low gate count devices such as SHM sensors. Thus, methods to reduce the power consumption of SHM sensors should focus on reducing the switching frequency, supply voltage, and gate count of the circuits. Assuming the frequency of operation can be adequately reduced, subthreshold circuit design can further decrease the power consumption of the digital blocks, such as an FFT processor [9,10].

In addition to DFT processing, an SHM sensor implementing impedance spectroscopy also requires analog circuits, such as an analog-to-digital converter (ADC), for measuring the system response to an excitation signal. Similar to the digital circuits, the power consumption of the ADC can be reduced by decreasing the bias voltage and the duty cycle of operation, resulting in a reduced sampling rate. For power-scalable ADC architectures with sampling rates in the Hz to kHz range of operation, the relationship between power consumption and sampling rate is approximately linear [11,12].

Undersampling, sampling at a frequency less than Nyquist, has been implemented in discrete component impedance spectroscopy circuits as a means for reducing the cost of the ADC and the power consumption of the ADC and DSP-FIFO circuits. Both single sine excitation systems [13,14] and mutlisine excitation systems [15,16] have been previously demonstrated. In comparison to single sine excitation, broadband excitation can be beneficial in reducing the total test time of impedance spectroscopy by simultaneously analyzing several frequency points, thereby decreasing the total settling time of the system [17]. Of the many types of broadband excitation signals, including pulse, chirp, pseudo-random binary sequences, and noise excitation, multisine signals enable straightforward specification of a line spectra excitation. This is helpful for analyzing frequency interactions resulting from aliasing due to undersampling.

Pioneering work performed by Creason and Smith [18,19] in the early 1970s with Nyquist sampled multisine signals recognized the benefit of using odd harmonic frequency distributions to prevent the corruption of excitation signals from even order system nonlinearities. Later work by Evans and Rees [20,21] introduced a new type of multisine distribution that is specifically designed to eliminate all nonlinear distortions, both even and odd, up to a specified order. However, neither of these approaches, nor other examples such as odd-odd or relative prime distributions, are specifically designed to prevent corruption of the excitation frequencies by nonlinear distortions when the output signal is undersampled. Therefore, these traditional multisine frequency distributions are not typically used when undersampling a system that includes a significant nonlinear component.