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An improved pendulum method for the determination of the center of gravity and inertia tensor for irregular-shaped bodies

Liang Tang [a], Wen-Bin Shangguan [b,*]

[a] School of Technology, Beijing Forestry University, Beijing 100083, PR China
[b] School of Mechanical and Automotive Engineering, South China University of Technology, Guangzhou 510641, PR China

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ABSTRACT

An improved Trifilar Torsional Pendulum (TTP) for the experimental determination of the Center of Gravity (C.G.) and inertia tensor (the three moments of inertia and the three products of inertia) for an irregular-shaped body is proposed and developed. In the improved apparatus, a universal joint is adopted to facilitate the adjustments of the C.G. of the body in line with the pendulum axis. To enhance the precision of the measurement, a tri-coordinate measuring machine is employed to measure the coordinates of the predefined points and vectors of axes, which are then used for calculating the C.G. and inertia tensor for an irregular-shaped body. The theoretical fundaments of an improved TTP, the experimental setup, data processing procedures, and error estimation for measuring C.G. and inertia tensor are presented. With the proposed TTP and data processing procedure, the relative error in the determination of the moments of inertia can be estimated within 1%, and the deviation of the measured C.G. to the theoretical C.G. is within 1.5 mm. An example is given to demonstrate the effectiveness of the new approaches.

1. Introduction

The Center of Gravity (C.G.) and inertia tensor (the three moments of inertia and the three products of inertia) are the prerequisite design parameters for designing the dynamic performances of a mechanical system, such as dynamic characteristics of automotive Powertrain Mounting System (PMS) by which the vehicle comfort is determined [1]. The methodologies for determining the C.G. and inertia tensor of an irregular-shaped body are divided into three categories: calculation method based on three-dimensional solid model [2], model parameter identification method [3,4], Trifilar Torsional Pendulum (TTP) method [5–9], and multi-cable pendulum [10].

It is a huge task to accomplish the three-dimensional solid modeling of an irregular-shaped body with all details, so the calculation method based on the CAD model is seldom used.

The methodology based on experimental modal analysis is simple in positioning postures of the body when identifying the C.G. and the inertia tensor, but with this method too many parameters need to be identified, the principle is complicate, the error analysis is difficult and the hardware requirements are expensive [8,12]. Moreover, the modal analysis method is sensitive to measurement noise, selection of response measurement points and excitation conditions [4]. Hence, this method is still seldom used in practical engineering.

However, TTP method with simple structure and theory, has been widely used in engineering field [5–7]. The shortcoming is that it requires the C.G. of the measured irregular-shaped body must coincide with the pendulum axis [5–7], and it is always a repeated and skilled task to make the C.G. of a complex body in line with the pendulum axis. In addition, for the published PPT apparatus [5–7], in order to compute the C.G. and inertia tensor of the powertrain, it is required to measure the distance between two predefined points, which is often measured by using a ruler or caliper, thus with low precision.

* Corresponding author.
E-mail address: shangguanwb99@tsinghua.org.cn (W.-B. Shangguan).

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To reduce the limitations of the current available methods [5–7], this paper proposes an improved TTP to determine the C.G. and the inertia tensor. Enhancements of the proposed TTP to the conventional TTP are: (1) a body with irregular-shaped is suspended under the TTP through an universal joint, thus the C.G. of the body can naturally lie on the pendulum axis of the TTP; (2) the methods for calculating C.G. and inertia tensor of the body are based on the coordinate and vector transformation.The coordinate and vector are measured with a tri-coordinate measuring machine and so with high precision. (3) The coordinates and vectors obtained in each measurement can be used simultaneously both for calculating the C.G. and the inertia tensor of an irregular-shaped body.

The errors of the proposed TTP come from two categories. One is from the precision of the experimental setup [13]. The length of the TTP wire and the empty mass of the TTP are two important parameters for the precision of the experimental setup. The method for identifying the length and the empty mass of the TTP are described and analyzed in detail. Another one is from data processing procedure for obtaining C.G. and inertia tensor of the body. To validate the effectiveness of the proposed procedure, the C.G. and inertia tensor for a regular body that merged with two different regular rectangular bodies are calculated with measured coordinates and vectors. The results are compared with the theoretical solution obtained from the CAD model. The comparisons show that the deviation between the measured C.G. and the theoretical C.G. is less than 1.5 mm, and the relative error for the measured moment of inertia around an axis is less than 1%.

Finally, the C.G. and the inertia tensor for an automotive powertrain (an engine plus a transmission) are measured with the developed TTP and the proposed data processing procedure by using two different approaches. One approach is to directly measure and calculate the parameters from the powertrain. Another approach is to calculate the parameters based on synthesizing the C.G. and the inertia tensor from the engine and the transmission, which are identified from the measurement on the engine and the transmission, respectively. The parameters from the two approaches agree reasonably well. It is demonstrated that the developed TTP and the data processing procedure has good characteristics for repeat measurement and with high precision. The experimental method, developed TTP and data processing technologies proposed in this paper can be used for getting the C.G. and the inertia tensor for an irregular-shaped body with high precision and less skill requirements.

2. Theoretical fundaments

2.1. Definition of coordinate systems

The inertial properties of a body consist of the mass, the Center of Gravity (C.G.), the moments of inertia and the products of inertia. Where, the moment of inertia and the products of inertia are the components of inertia tensor. The mass is unrelated to any coordinate system and can be measured easily. But it is a rather arduous task to determine of the C.G. and the inertia tensor. In addition, the C.G. and the inertia tensor should be defined in one specific coordinate system. To calculate the above parameters, the following coordinate systems are defined and used in the study.

(1) Ground coordinate system (O_G–X_G,Y_GZ_G)
The ground coordinate system (O_G–$X_GY_GZ_G$) is established with its origin located on the ground as shown in Fig. 1a. The coordinates of a point or a vector of an axis measured from a tri-coordinate measuring machine is defined in this coordinate system. The coordinates and vectors are the basic data for obtaining the C.G. and inertia tensor. The advantage for the tri-coordinate measuring machine is the high precision of the measured coordinates and vectors of the target body.

(2) Reference coordinate system (O_R–$X_RY_RZ_R$)
The second coordinate system, referred to Reference coordinate system (OR–$X_RY_RZ_R$), is used to describe the C.G. of an irregular-shaped body and is shown in Fig. 1b. The coordinate system is fixed to the body. The origin and the direction of each coordinate axis of the Reference coordinate system are defined with a tri-coordinate measuring machine.

(3) Centroid coordinate system (O_C–$X_CY_CZ_C$)
Moving the origin of the Reference coordinate system to the C.G. of the measured body, the Centroid coordinate system is obtained and is shown in Fig. 1c. The direction of each axis (O_CX_C, O_CY_C, O_CZ_C) in Centroid coordinate system is the same as that in Reference coordinate system O_R–$X_RY_RZ_R$.
The inertia tensor of the measured body is defined in Centroid coordinate system.

(4) On-board coordinate system (A–$X_AY_AZ_A$)
The On-board coordinate system, A–$X_AY_AZ_A$, as shown in Fig. 1d, is fixed to the measured body and defined by three reference points A, B and C. The three referential points should not align in a straight line. The selection of the three points is based on the convenience that the coordinate of the three points can be easily measured with the tri-coordinate measuring machine. The origin of On-board coordinate system is located at A, X-axis is defined by vector $\overrightarrow{AB}$, and Z-axis is determined by vector $\overrightarrow{AB} \times \overrightarrow{AC}$. This coordinate system is used as the transition coordinate system for the subsequent data processing.

It is seen that the Reference coordinate system, the Centroid coordinate system and the On-board coordinate system are all attached to the measured body. That is to say, the coordinates of a predefined point and a vector for an axis on the body with respect to these three coordinate will not change whatever the suspending posture of a body is.

2.2. Method for obtaining moment of inertia of a body along an axis

The moment of inertia of an irregular-shaped body around an axis can be obtained by using a TTP as shown in Fig. 2. Departing the lower disk from its equilibrium position leads in an oscillation with period T. Given by small