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Pulse width influence in fast frequency measurements using rational approximations

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ABSTRACT

Many applications have been proposed for the frequency measurement principle using rational approximations; every measurement has an error value that is related to the resolution of the measuring tool; in this technique, measuring is done by comparing an unknown frequency and a known frequency signals. Theoretically, the error in measurement in the mathematical model of this principle is limited only by repeatability of the standard. But in practice, for arbitrary combination of unknown and reference signal, the pulse width of the signals can modify the method application; In this work a criterion for choosing the optimal pulse width values is defined, by investigating the relationships between the reference signal's period and the pulse width.

1. Introduction

The frequency measurement principle by rational approximations [1] has many outstanding properties, namely invariance to jitter and resolution limited just by stability of the reference oscillator [2,3]. This technique carries a comparison between a unknown frequency signal (measurand) and a known frequency signal (reference). One of the most important theoretical assumptions in such comparisons is that both signals need to have the same pulses shape in both compared trains (among them, one of the most critical is pulse width). Unfortunately, in real life it is difficult to fulfil such condition. So, the main scope of the present work is to research how the imperfections of real pulses can affect the theoretical method of [1–3]. Metrology of time has many applications in many areas of science and technology, in particular fast and accurate frequency measurements are needed when dealing with highly sensitive sensors used for detection of specific chemical compounds [4–7]. Due to the characteristics of rational approximations principle, this method could be used for such task [8,9].

In this work, the basic theory used in fast frequency measurements by the principle of rational approximations is expanded; this is done by analyzing the effect of the

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width in the reference and measurand. The aim of this research is to define a criterion for choosing the optimal value of the pulse width, by doing so, the optimal signal-conditioning parameters could be determined and best measurements could be achieved.

2. The frequency measurement principle by rational approximations

Since this measurement technique was proposed [1], many applications and analysis have been carried on; in this section, for clarity, the basics of this principle will be reviewed.

Let it two signals measurand (S_x) and reference (S_0) be compared using a logical condition AND, both signals have the same pulse width duration (τ) and distinct periods (T_x and T_0 respectively). A coincident pulse train ($S_x\&S_0$) is generated. After the first coincidence, the counting of pulses in S_x and S_0 starts. P_n and Q_n are the number of pulses in S_x and S_0 respectively, n denotes the number of coincidence. From now on, where a coincidence exists, a n-fraction (P_n/Q_n) is generated (Fig. 1).

When two perfect coincidences exist, a mediant fraction is formed, and the condition of Eq. (1) is maintained.

$$\frac{P_{n+i}}{Q_{n+i}} < \frac{P_{n+i}+P_n}{Q_{n+i}+Q_n} < \frac{P_n}{Q_n} \tag{1}$$

The measurement time (M_t), since the beginning until the n-th fraction is given by:

$$M_t = Q_n T_0 = Q_n \frac{1}{f_0}. \tag{2}$$

As consequence of a continuous counting process, n increments until $n+i$, where $n,i>0$ and $n,i \in N$. N denotes the natural numbers conjunct. P_{n+i} and Q_{n+i} appear after P_n and Q_n, the $n+i$ value increases as result of time measurement until a number m where the best approximation is obtained, this is another well known property in classic metrology, by increasing measurement time, the accuracy of measurement increases [10]. Finally the unknown frequency value (f_x) of S_x can be obtained using the known frequency value (f_0) of S_0:

$$f_x = f_0 \frac{P_n}{Q_n}, \tag{3}$$

from Eqs. (1) and (2):

$$f_x = f_0 \frac{\sum_m P_n}{\sum_m Q_n}. \tag{4}$$

According to [11,12], the measurement process stops when the condition of 1 with zeros is achieved.

Any frequency value can be approximated using Eq. (4), just by counting the pulses in S_x and S_0. This approach has been investigated in previous works [1,13,14,2,3,15], and it allows with simple calculations, in a very fast way to know the measurand value. From the other hand, it is obvious from Fig. 1 that the width of the perfect coincidence (appears in Fig. 1 as most wide blue[1] squares) strongly depends on the equality of coincident pulses. In [2] it is shown that position of such a perfect coincidence is always on the same place (for the same pair of S_0 and S_x) and does not depend on such natural phenomena like jitter. But it is evident from same Fig. 1 that other bad thing which can destroy the geometry rational approximation method is when the pulses of independent trains S_0 and S_x will have no the same duration along time axis, or simply pulse width. Further considerations and simulations herein provided was done with the aim to detect limits when the difference of pulse widths S_0 and S_x can destroy the geometric relations between trains S_0 and S_x as presented in Fig. 1; or in simple words to discard the use of this frequency measurement method.

3. Pulse width analysis

The duration of the pulses in $S_x\&S_0$ is defined as the time of coincidence (t_{0x}) between a pulse in S_x and S_0. The value of t_{0x} and the number of pulses that exists in $S_x\&S_0$ depends of the values of f_x and f_0, also of the relationship of τ and T_0. Basing on analysis of variation of t_{0x} (durations of blue pulses in Fig. 2) we can conclude that in this method this parameter is strongly variable. In general case, its variation is sub ordered by graphical distribution of Fig. 4c. However, in practice due to many physical factors such distribution never repeats in the same way; this circumstance was the principal mean for deeper research of t_{0x} behavior reflected later on all nine cases of Fig. 5. As it was shown on previous works [9], theoretically the error in the measurement is just defined by the duration on the coincidence of τ. In fact from the existence of such coincidence, the pulse width should obey to the condition shown in Eq. (5), otherwise it will be a time when two pulses in any signal would be coincident with one pulse in the other signal.

$$\tau < \frac{1}{2} T_0 \tag{5}$$

In Fig. 1, the duration of coincidences of pulses in signal comparison is shown. Defining t_{0x} as the lasting time of the coincidence:

$$t_{0x} = \tau \pm \Delta\tau, \tag{6}$$

where $\Delta\tau$ is the difference between a two coincident pulses, it can be positive or negative according the pulses position, the maximum value that it can take is [15]:

$$\Delta\tau = 2\tau. \tag{7}$$

Eq. (5) is a necessary and sufficient condition for application of the frequency measurement principle by rational approximations. When coincidence of pulses in two signals comparison exists, the following cases could exist:

- A perfect coincidence exists when two pulses are perfectly coincident ($\Delta\tau = 0$):

$$t_{0x} = \tau. \tag{8}$$

- If S_0 pulses are ahead of the pulses in S_x, a partial coincidence exists; all the pulses in S_0 and S_x have a different time when they start. The time when a pulse in S_0 starts is defined by $t_{0Q_n} = Q_n T_0$ (the measurement time where

[1] For interpretation of color in Figs. 1 and 2, the reader is referred to the web version of this article.