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Comparison of chaos optimization functions for performance improvement of fitting of non-linear geometries

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A B S T R A C T

Fitting algorithms play an important role in the whole measuring cycle in order to derive a measurement result. They involve associating substitute geometry to a point cloud obtained by an instrument. This situation is more difficult in the case of non-linear geometry fitting since iterative method should be used. This article addresses this problem. Three geometries are selected as relevant case studies: circle, sphere and cylinder. This selection is based on their frequent use in real applications; for example, cylinder is a relevant geometry of an assembly feature such as pin-hole relationship, and spherical geometry is often found as reference geometry in high precision artifacts and mechanisms.

In this article, the use of Chaos optimization (CO) to improve the initial solution to feed the iterative Levenberg–Marquardt (LM) algorithm to fit non-linear geometries is considered. A previous paper has shown the performance of this combination in improving the fitting of both complete and incomplete geometries. This article focuses on the comparison of the efficiency of different one-dimensional maps of CO. This study shows that, in general, logistic-map function provides the best solution compared to other types of one-dimensional functions. Finally, case studies on hemispheres and industrial cylinders fitting are presented.

1. Introduction

Metrology for geometrical entities has an important role in manufacturing. It is the procedure to verify geometric attributes of products and is most often realized by coordinate metrology [1]. In coordinate metrology, least square (LS) fitting to associate substitute geometry to point cloud is a fundamental step to derive the measurement result [2–5]. By applying this procedure, dimensional measurements, such as the diameter of a circle or sphere, the angle between two lines, the distance between two spheres, and geometrical measurements, such as flatness, roundness, parallelism, coaxiality deviations, are performed accurately and flexibly. Therefore, the fitting algorithm is a fundamental element in the whole chain of dimensional metrology. Based on Hoppe et al. [6], fitting the substitute geometry is called a "function reconstruction", since it is an association of points to a defined function. From the point of view of the fitting algorithm, basic geometries can be divided in two groups: linear and non-linear. Linear geometries include 2D line and plane. Most other types of basic geometries, such as circle, sphere, cylinder, and cone are considered non-linear geometries.

The fitting of linear geometries can be analytically solved and a single global optimal result obtained. In the case of non-linear geometries fitting, an iterative procedure, minimizing some objective function, is used instead. The lack of prior knowledge about the parameters of the geometry to fit significantly increases the difficulties in

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the fitting itself. Consequently, for non-linear geometric fitting an initial solution must be selected to initialize the iterative procedure. This initial solution should be good enough to ease the search algorithm in the estimation of the best fitting geometry. The situation in which the prior knowledge about the geometry to be fitted is not known can be found in reverse modeling. One example is that it is very difficult to identify the axis of direction of a cylinder when prior knowledge about the cylinder is unknown. Not only accuracy problem in the fitting process but also cost of computation is relevant for this situation. The reason is that current measurement instruments, especially the non-contact one, are able to capture thousands or even millions of points within short period of time. As such, time required for the fitting becomes relevant issues as it increases the overall measuring time. Hence, fast and accurate fitting is needed to realize a high-speed inspection to reduce inspection cost, and hence reducing the production cost [7]. Hence, not only an accurate, but also a high-speed fitting procedure is required.

In a previous paper [8] the authors presented the efficiency of non-linear fitting by utilizing Chaos Optimization (CO) method to select the initial point for the LM iterative algorithm (Chaos-LM algorithm). Results of the study show that CO improves the fitting accuracy without scarifying the computation time [8]. In this paper, comparison among CO functions for the LS fitting of non-linear geometries by Chaos-LM algorithm are presented. The purpose is to study the performance of CO method with respect to its different types of continuous one-dimensional maps in different fitting situation and to identify the most effective for most cases. The paper is structured as follows. In Section 2, the various one-dimensional maps are introduced. Comparison among one-dimensional maps and case studies are presented in Section 3. Finally, concluding remark and future developments are proposed in Section 4.

2. Chaos theory and one-dimensional map functions

The defining parameters of geometries define them as either linear or non-linear. Line and Plane are classified as linear geometry since their defining parameters are linear. Other basic geometries have non-linear parameters that define their shapes. They are circle, sphere, cylinder, cone, and torus. Therefore, they are named "non-linear geometries". This paper addresses the problem of non-linear Least Squares (LS) fitting circle, sphere, and cylinder. Circular and spherical geometries have many applications, for example the sphere, as it univocally defines a single point (its center) is often adopted as reference geometry [9,10] and also commonly adopted as artifact the verification of the performance of metrology instruments [11,12]. Many rotational part found in mechanical product are constituted by cylindrical features, such as shafts and holes. Moreover, a pin-hole is the most common assembly feature, and it is represented by a couple of mating cylinders [13].

To solve the problem of fitting non-linear geometries, the well-known Levenberg–Marquardt (LM) algorithm can be used [14,15]. The LM algorithm strongly depends on an initial solution $\mathbf{p}_0$ that must be provided [16]. The choice of the initial solution is critical, because the function to optimize is multi-modal: many local optima and high curvature characterize it. Therefore, the search can get trapped locally in a non-global optimum. The probability of getting trapped highly depends on the staring solution [16]. Jenecki et al. [17] addressed a very specific problem: in the case of bearings, geometries are not exactly cylindrical but present saddle and barrel geometries. They developed a mathematical model for these geometries, but then noticed that the Gauss–Newton algorithm adopted for the fitting was very sensitive to the initial solution. Therefore, they developed a specific methodology for the seeding of the optimization algorithm. In a previous paper by the authors of the present paper [8], the concept of non-linear fitting and the Levenberg–Marquardt algorithm as well as the problem of seeding initial points were introduced. Then, a Chaos-Levenberg–Marquardt (Chaos-LM) approach to the non-linear least square optimization was introduced. In that work, a logistic one-dimensional map function was introduced and adopted. This work wonders if the choice of the logistic map was optimal: more competing one-dimensional maps are to be introduced. The reader is kindly asked to refer to the mentioned paper for detail about the non-linear fitting, LM algorithm, and Chaos-LM optimization. Here, just the considered one-dimensional maps, for comparison, will be introduced.

Chaos is a semi-random behavior generated by a non-linear function. It creates a chaotic dynamic step which can be useful to escape from local optima region in a search process. It is deterministic because each step can be uniquely determined from the previous step; however, though deterministic, the step will realize a hardly predictable path. Hence, it is similar to the observation of a stochastic process. The concept differs from the improved heuristic searches (random-based algorithm) which work based on rejection-accepting probability test [28]. Since chaos dynamic is not stochastic, it differs from heuristic search. Searching through regularity of chaotic motion is its fundamental recipe [29]. This motion represents a dynamical trajectory system motion that can be represented as mapping of one variable to other variable. Chaos function significantly depends on its initial condition in the sense that two initial values, which can be spatially very close to each other, when subject to chaotic step trajectories will diverge at exponential rate. Considering a mapping of $\mathbf{t_{k+1}} = \mathbf{F_M}(\mathbf{t_k})$, which is n-dimensional variables, the significant difference of the chaos trajectory $\mathbf{t_{k+1}}$ with respect to two initial conditions $\mathbf{t}_0$ and $\mathbf{t}_{0+\Delta}$, can be modeled by a Lyapunov exponent as [18]:

$$\left|\mathbf{F}_M^n(\mathbf{t}_0) - \mathbf{F}_M^n(\mathbf{t}_{0+\Delta})\right| = \Delta e^{n\lambda(\mathbf{t}_0)} \tag{1}$$

where n is number of iteration and the Lyapunov exponent is represented by the $\lambda(\mathbf{t}_0)$ function. A function, to have a chaotic behavior, should have dimension $\geqslant 3$ [23]. Otherwise, this behavior can be observed in one-dimensional functions if the map is not invertible. A map $\mathbf{F_M}$ is not invertible, if and only if, given $\mathbf{t_{k+1}}$, we cannot solve $\mathbf{t_{k+1}} = \mathbf{F_M}(\mathbf{t_k})$ for $\mathbf{t_k}$. In this case, the solution of $\mathbf{t_k} = \mathbf{F_M^{-1}}(\mathbf{t_{k+1}})$ does not exist. This is due to one single value