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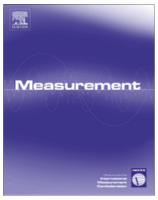

Tension determination of stay cable or external tendon with complicated constraints using multiple vibration measurements

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ABSTRACT

A novel concept of combining the mode shape ratios with the modal frequencies was recently introduced to develop an accurate method for the determination of stay cable force. A crucial restriction of symmetric boundary constraints at both ends was imposed in this method for the optimization of effective vibration length. However, there certainly exist a number of cases with apparent unsymmetrical boundary constraints such as the case of stay cables with supplementary dampers and that of external tendons affected by intermediate diaphragms. To deal with such difficulties, this method is further generalized in the current study by introducing a shifting parameter in the shape function to effectively consider the unsymmetrical boundary constraints. The numerical difficulty of the more complicated nonlinear optimization process associated with this new formulation is first discussed, followed by extensive numerical investigations, practical verifications, and real applications.

1. Introduction

An accurate estimation of stay cable force typically plays an important role in the health monitoring of cable-supported bridges. Similar significance also applies to the tension determination of external tendons generally used in the prestressing of box girder bridges [1,2]. Several devices such as hydraulic jacks, strain gauges, load cells, and embedded fiber Bragg grating (FBG) sensors have been previously adopted in engineering practice to evaluate the tension of stay cable or external tendon. These permanent devices usually suffer either from questionable accuracy, high cost, complicated fabrication, or deterioration with time to obstruct their popular applications. Based on the fact that the magnetic permeability of steel material is sensitive to its existing stress, elasto-magnetic (EM) sensors have also been recently developed to conduct the tension measurement in numerous cases [3–7]. Delicate calibration tests in the laboratory or field, however, are normally required for this EM technology.

Due to its easy operation and wide applicability, the ambient vibration method is more commonly employed for monitoring the tension of stay cable or external tendon than the static approaches mentioned above in either the construction [2,8,9] or service stage [10–13]. This method is regularly applied by first identifying the cable frequencies from the ambient vibration measurements with conventional contact sensors or advanced non-contact sensors [14–18]. A pre-determined formula or numerical simulation is then used to estimate the tension. The ambient vibration method was initially implemented according to the string theory where the stay cable or external tendon is simply modeled as a transversely vibrating string with hinged boundary conditions to obtain an analytical formula merely requiring given vibration length and mass per unit length. Later studies tried to improve the accuracy

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of this method by further incorporating the effects of flexural rigidity [19–23], gravity sag [24–26], and complicated boundary conditions [27–31] with more involved analytical or empirical formulas. Alternative numerical approaches have also been attempted with finite element (FE) analysis to search for the optimal values of tension, flexural rigidity, and other parameters such that all the identified modal frequencies can be best fitted [12,32–35].

Aside from the efforts for a more comprehensive modeling, the careful selection of appropriate parameter values to faithfully reflect the actual vibration behavior is probably of equal importance in improving the accuracy of the ambient vibration method. In practical applications, rubber constraints and special anchorage systems are usually installed near both ends of stay cables. As for the case of external tendons, intermediate diaphragms are frequently utilized to change the direction of tendon for providing an optimal prestressing inside the box girder. These designs undoubtedly increase the uncertainty of boundary conditions and blur the selection of effective vibration length, which is a particularly sensitive parameter to determine the tension. Consequently, it is generally a challenging task to accurately decide the effective vibration length required for determining the tension with a given formula. Even for the numerical approach where the length of cable or tendon, rubber constraints, and intermediate diaphragms can be directly modeled in the corresponding FE analysis, it is still necessary to simulate the rubber constraints with effective elastic parameters or the detailed contacts between the tendons and diaphragms. But unfortunately, these parameters cannot be conveniently assessed in practice.

Aimed to more effectively tackle the modeling and parameter issues encountered in better estimating the tension, a novel concept of combining the mode shape ratios with the modal frequencies was recently introduced by the authors to develop a convenient and accurate method for the determination of stay cable force [36–38]. Multiple synchronized vibration signals of a stay cable [18] were first processed to obtain the mode shape ratios at various sensor locations for each observable mode. These ratios were then compared with the sinusoidal mode shapes based on the simply-supported beam model with axial tension to independently obtain an optimal effective vibration length such that the total squares error for all the considered modes is minimized. Other researchers have also started to explore similar concept in these two years [39–41]. With this length obtained, the cable force and flexural rigidity can subsequently be solved by simple linear regression techniques using the identified cable frequencies and the analytical formula. In this method, a key issue in the optimization process of effective vibration length was to describe the sensor locations by selecting the pre-known middle point of cable as the reference origin point for the sinusoidal shape functions. With this choice, it is equivalent to assume the symmetry of mode shape functions with respect to the middle point of cable. In other words, a crucial restriction of practically symmetric boundary constraints at both ends was imposed with this formulation.

Even though the cable anchorage systems in most of the practical designs may not be far away from the symmetric simplification, there certainly exist a number of cases with apparent unsymmetrical boundary constraints, especially when supplementary dampers are installed at the deck ends of stay cables. In addition, different types of deviator used at the intermediate diaphragms in the middle of a box girder to change the line direction of external tendons may induce more intricate constraints. To deal with such difficulties, the recently developed method is further generalized in the current study by introducing an extra shifting parameter of origin in the sinusoidal shape function to effectively consider the unsymmetrical boundary constraints. The numerical difficulty of the more complicated and sensitive nonlinear optimization process associated with this new formulation is first discussed in this paper, followed by extensive numerical investigations, practical verifications, and real applications.

2. Methodology based on determination of effective vibration length

A stay cable system is usually composed of three parts: (1) a free length section in the middle; (2) two anchorage zones at both ends; and (3) two transition zones between the previous two parts. The combination of the anchorage zone and the transition zone is typically called the cable anchorage device. In general, flexible rubber constraints are installed at the front end of anchorage device to decrease the bending stress at anchorage ends, centralize the cable, and moderate the fatigue problem. Due to the complicated anchorage device whose detailed design varies with the suppliers, it is difficult to truthfully define the boundary conditions and successfully model the sections close to both ends in performing the cable analysis. As for the case of external tendon, the intermediate diaphragms inside the box girder to alter the direction of tendons can also create very complicated situations. Even so, it has been demonstrated that the effect of anchorage device on the cable vibration is limited to a finite range near the anchorage ends [36–38]. The effect of intermediate diaphragms on the tendon vibration is expected to bear similar feature. Thus, the primary free length section in the middle of cable or tendon should be eligibly modeled by a simply supported beam with an axial tension. The only crucial problem left is how to determine an effective length for this model.

2.1. Simply supported beam model with an axial tension

Considering a simply supported beam subjected to an axial tension T, an analytical formula for the modal frequencies of this model can be solved as:

$$\left(\frac{f_k}{k}\right)^2 = \frac{T + \frac{k^2\pi^2 EI}{L^2}}{4\bar{m}L^2} = \frac{\frac{T}{L^2} + \frac{k^2\pi^2 EI}{L^4}}{4\bar{m}} \text{ or } T = 4\bar{m}L^2\left(\frac{f_k}{k}\right)^2 - \frac{k^2\pi^2 EI}{L^2}, \quad k = 1, 2, 3, \ldots \tag{1}$$

where L signifies the beam length, $\bar{m}$ symbolizes the mass per unit length, E denotes the Young's modulus, I