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On-line dynamic error compensation of accelerometers by uncertainty-optimal filtering

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ABSTRACT

The output signal of an accelerometer typically contains dynamic errors when a broadband acceleration is applied. In order to determine the applied acceleration, post-processing of the accelerometer's output signal is required. To this end, we propose the application of a digital FIR filter. We evaluate the uncertainty associated with the filtered output signal and give explicit formulae which allow for on-line calculation. In this way, estimation of the applied acceleration and the calculation of associated uncertainties may be carried out during the measurement. The resulting uncertainties can strongly depend on the design of the applied filter and we describe a simple method to construct an uncertainty-optimal filter. The benefit of the proposed procedures is illustrated by means of simulated measurements.

1. Introduction

Accelerometers are electromechanical transducers whose dynamic behavior can for certain amplitude and frequency ranges be described in terms of a linear time-invariant (LTI) system [1]. Often, a second-order model is appropriate [2], whose frequency response can be determined by sinusoidal excitations [3]. One goal in the construction of an accelerometer is to provide a frequency response with constant magnitude and linear phase over a large range of frequencies. When the spectrum of the applied acceleration is restricted to this frequency range, the accelerometer's output signal is – up to a time shift – proportional to the applied acceleration. However, for larger frequencies such an ideal behavior is often not met. The accelerometer's output signal then contains dynamic errors such as ringing. In order to determine the applied acceleration from these dynamic measurements, appropriate tools from digital signal processing (DSP) need to be employed. As the number of dynamic measurements in metrology is increasing in recent times [2–6] there is a growing interest among metrologists in these techniques.

For systems of LTI type the physical quantity of interest is determined by a deconvolution of the measured sensor output signal [7]. Digital filtering is an appropriate tool to carry out this deconvolution and the design of appropriate compensation filters is well-known in DSP, see e.g., [8–13]. Successful application of these techniques has already been demonstrated in several examples from metrology [4,6,14–16]. One important aspect in metrology is the evaluation of the uncertainty associated with the result of a measurement for which particular guidelines are available [18,19]. These guidelines do not address the case of dynamic measurements, and recently proposals have been made to extend these guidelines to dynamic measurements [4,6,14–17].

In this paper we propose digital filtering to the output signal of an accelerometer in order to determine the applied acceleration, and we employ the recently proposed uncertainty evaluation scheme [14,15] to calculate the uncertainty associated with the obtained estimate of the acceleration. The focus is to obtain a calculation scheme which allows for an on-line analysis. To this end, we design the digital filter as a cascade of an FIR compensation filter

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whose frequency response approximates the reciprocal of the frequency response of the accelerometer [8] and a FIR linear-phase lowpass filter. As accelerometers can usually be described by a second-order model [2], a moderate order of the FIR compensation filter is sufficient. The digital filter introduces a time shift of the estimated acceleration, but we show that this time shift is typically small compared to the length of the impulse response of the accelerometer. We give explicit formulae for the calculation of uncertainties which allow for on-line calculation. Hence, both, the estimated acceleration and its associated uncertainty, can be calculated in real-time.

It turns out that the resulting uncertainties strongly depend on the design of the employed FIR filter and a further goal of the paper is to design an uncertainty-optimum filter. Optimal filters such as, e.g., the Wiener filter, are well-known in DSP (e.g., [20,21]). However, rather than employing the criteria underlying the construction of these filters we address the resulting uncertainty as the criterion relevant in metrological applications. We show that by a simple adaptation of the lowpass filter the resulting uncertainty may easily be approximately minimized and that this can lead to a significant reduction in uncertainty.

This paper is organized as follows. In Section 2 we state the underlying assumptions and introduce the employed model for the accelerometer. Digital filtering for dynamic error compensation is then briefly discussed in Section 3. In Section 4 we describe the proposed uncertainty evaluation scheme and the construction of an uncertainty-optimal filter. The benefit of the proposed procedures is finally illustrated by analyzing simulated data in Section 5.

2. Accelerometer model

The input–output behavior of an accelerometer can for a certain amplitude and frequency range be modeled by the differential equation

$$\ddot{x}(t) + 2\delta\omega_0\dot{x}(t) + \omega_0^2 x(t) = \rho a(t), \tag{1}$$

cf. [2]. The output signal $x(t)$ of the accelerometer is passed through a charge amplifier and undergoes an analogue-to-digital conversion (ADC). We model these ADC errors as additive noise, see Fig. 1. The frequency response of a charge amplifier is usually flat and shows a linear phase within the operating range. It thus essentially leads to a time shift in the output signal. When the dynamic behavior of the charge amplifier is more involved, its dynamics would need to be accounted for by constructing an additional compensation filter. Here we assume that the charge amplifier is perfectly flat and we ignore the time shift introduced by its linear phase.

We assume in addition that an upper bound $\bar{A}(\omega)$ on the magnitude spectrum of the input acceleration is available.

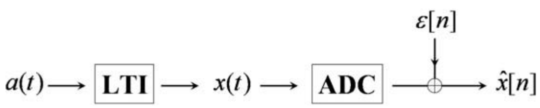

Fig. 1. Scheme of an accelerometer measurement with subsequent analogue-to-digital conversion.

This bound is employed for a reliable uncertainty evaluation. Such a bound may be derived from prior knowledge about the input signals or from related measurements.

3. Digital compensation filter

We consider the application of an FIR digital compensation filter $g(z)$ to the discrete-time output signal $x[n]$ of the accelerometer for discrete-time estimation of the applied acceleration $a[n] = a(n \cdot T_s)$, where $f_s = 1/T_s$ denotes the sampling frequency. The compensation filter should be a good approximation to the reciprocal frequency response of the sensor in the frequency region where the input signal has significant content while it should allow at the same time for noise attenuation at higher frequency regions. The filter is constructed as a cascade of a deconvolution filter $g_{deconv}(z)$ and a lowpass filter $g_{low}(z)$. The approximation to the reciprocal frequency response of the sensor is provided by the deconvolution filter, while the lowpass filter controls the tradeoff between a broad passband in the operating range and noise attenuation in the high-frequency region.

The filter $g(z)$ is built in two steps. First, the deconvolution filter is constructed by a least-squares fit of its frequency response to the reciprocal frequency response $H(j\omega)$ of model (1). In a second step, the lowpass filter is chosen, see also Section 4. Since the coefficients of the model are not known exactly, the coefficients of the filter $g_{deconv}(z)$ derived from them are uncertain as well and so is $g(z)$. We indicate this by $\hat{g}(z)$. Estimates $\hat{a}[n]$ of the applied acceleration are then obtained from the available accelerometer output signal $\hat{x}[n]$ through

$$\hat{a}[n - n_0] = (\hat{g} * \hat{x})[n], \tag{2}$$

where n_0 accounts for a possible (small) time delay introduced for the construction of the compensation filter. For further details cf. [6].

4. Uncertainty evaluation

The uncertainty associated with the estimates (2) is given by

$$u^2(\hat{a}[n - n_0]) = u^2((\hat{g} * \hat{x})\,[n]) + \gamma^2/3. \tag{3}$$

The first term on the right-hand side accounts for the uncertainty of the coefficients of the compensation filter and for unsuppressed noise. The second term considers remaining dynamic errors caused by imperfect compensation and it results from the following approximate bound on the dynamic error $\Delta[n]$:

$$|\Delta[n]| \lesssim \frac{1}{2\pi}\int_{-\pi f_s}^{\pi f_s} |e^{j\omega n_0/f_s} G(e^{j\omega/f_s}) H(j\omega) - 1| \cdot \bar{A}(\omega)\mathrm{d}\omega =: \gamma, \tag{4}$$

which can be derived using Fourier techniques [14,16]. In (4) $G(e^{j\omega/f_s})$ denotes the frequency response of the compensation filter, and $\bar{A}(\omega)$ the upper bound on the magnitude spectrum of $a(t)$. By assigning a uniform probability density function for the dynamic error within the bounds