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Technical note

Frequency estimation of discrete time signals based on fast iterative algorithm

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ABSTRACT

A fast iterative algorithm for frequency estimation is developed in this paper to improve the frequency tracking performance. If the signal is transformed by a mathematical tool, the signal to noise ratio (SNR) should not be greatly reduced after the transformation. The analysis presented in this paper showed that the traditional method for frequency estimation causes large noise at high frequency range, therefore, the suitable estimation range of traditional method is only from 0 to $fs/6$ Hz (fs is the sample frequency). In order to overcome this limitation, a new structure of iterative algorithm is established to extend the upper bound frequency from $fs/6$ to $fs/2$ Hz. The experimental noisy sinusoid signal frequency estimation and chirp signal frequency tracking confirmed that the novel algorithm showed improved performance. Furthermore, the average estimation error was decreased over 30% (under SNR = 15 dB) when applying the novel iterative algorithm. The novel iterative algorithm will have broad applications in fields of signal processing and communication systems.

1. Introduction

The adaptive iterative algorithm has been proved to be a simple and effective tool to estimate frequencies in discrete-time signal analysis. This type of algorithm has been successfully used in the fields of speech signal processing [1–3] and communication [4,5] due to the good performance of frequency estimation and frequency tracking. However, the iterative algorithms that are currently used have limitations in practical applications. The precision of frequency estimation will be degraded when signal is polluted by the broadband noise. The algorithms will even become invalid when a DC noise is involved.

Traditional adaptive iterative algorithms used in frequency tracking tend to generate large noise and distortion at high frequency range, resulting in narrow frequency tracking ranges. Generally, the signal to noise ratio (SNR) should not decrease over 3 dB after mathematical transformation. Based on this requirement, the suitable frequency estimation range for the traditional algorithms is only from 0 to $fs/6$ (fs is the sample frequency). In order to overcome this limitation, a fast iterative algorithm was developed in this paper. This novel algorithm used more discrete time signal samples. The differential function was applied to remove the interference of the constant noise and extend the upper bound frequency from $fs/6$ to $fs/2$. This algorithm was able to track the discrete time signal frequency from 0 Hz to $fs/2$ Hz with improved precision. The experiment results confirmed the improved performance of the novel iterative algorithm in signal frequency estimation and chirp frequency tracking. This novel algorithm can be used

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in high resolution signal processing and communication systems.

The arrangement of the full text is as follows. The next section is to analyse the limitation of traditional algorithms. The third section presents the development of the novel algorithm. The convergence of the new algorithm is introduced in the fourth section. Experiment results are described in the fifth section. The conclusions are drawn finally.

2. The limitation of traditional algorithms

The standard discrete time sinusoid signal can be written in Eq. (1).

$$s(n) = A\sin\left(2\pi\frac{f}{fs}n+\theta\right) \tag{1}$$

Here fs is the sample frequency, A is the amplitude, θ is the phase, $n = 1, 2, 3, \ldots$ and $0 \leqslant f \leqslant fs/2$. When the signal is polluted by noise $\varepsilon(n)$, the above equation can be written as Eq. (2). And Eq. (3) can be deduced by Eq. (2).

$$y(n) = s(n) + \varepsilon(n) = A\sin\left(2\pi\frac{f}{fs}n+\theta\right) + \varepsilon(n) \tag{2}$$

$$\begin{aligned} y(n+1)+y(n-1) &= A\sin\left(2\pi\frac{f}{fs}n+\theta+2\pi\frac{f}{fs}\right) + A\sin\left(2\pi\frac{f}{fs}n+\theta-2\pi\frac{f}{fs}\right) \\ &\quad + \varepsilon(n+1)+\varepsilon(n-1) \\ &= 2A\sin\left(2\pi\frac{f}{fs}n+\theta\right)\cos\left(2\pi\frac{f}{fs}\right) + \varepsilon(n+1)+\varepsilon(n-1) \\ &= 2\cos\left(2\pi\frac{f}{fs}\right)s(n) + \varepsilon(n+1)+\varepsilon(n-1) \end{aligned} \tag{3}$$

If the $\varepsilon(n)$ is very small, Eq. (4) can be calculated from Eq. (3).

$$\cos\left(2\pi\frac{f}{fs}\right) \approx \frac{y(n+1)+y(n-1)}{2y(n)} \tag{4}$$

According to the references [1,4], the lattice notch filter can be constructed by an iterative algorithm. Then based on the recursive least square (RLS) principle, the instant frequency can be estimated by the Eqs. (5) and (6). They can be regarded as the traditional algorithm for frequency detection.

$$\begin{aligned} k(n) &= -C(n)/D(n) \\ C(n) &= \lambda C(n-1) + (1-\lambda)y(n-1)[y(n)+y(n-2)] \\ D(n) &= \lambda D(n-1) + (1-\lambda)2y(n-1)^2 \\ k(n) &= \gamma k(n-1) + (1-\gamma)k(n) \end{aligned} \tag{5}$$

Here, λ is the forgetting factor, and γ is the smoothing factor.

Then the instant frequency $f(n)$ of every signal sample can be calculated as Eq. (6).

$$f(n) = fs \times arc\ \cos(-k(n))/2\pi \tag{6}$$

Although these equations are simple to be solved, there are some problems that have not been taken into account. First, if the noise $\varepsilon(n)$ is a constant in Eq. (2), this traditional method may be invalid. Second, if the $\varepsilon(n)$ is the zero mean random noise, this method is not suitable for high frequency ($f > fs/6$) estimation based on the following derivation.

The SNR should not significantly change between the both sides of Eq. (3). Thus the SNR at the left side can be calculated in Eq. (7). The Var is referred as the variance. The SNR of right side can be written in Eq. (8).

$$\begin{aligned} & SNR\{y(n+1)+y(n-1)\} \\ &= \left\{\frac{Var\left[A\sin\left(2\pi\frac{f}{fs}n+\theta+2\pi\frac{f}{fs}\right) + A\sin\left(2\pi\frac{f}{fs}n+\theta-2\pi\frac{f}{fs}\right)\right]}{Var[\varepsilon(n+1)+\varepsilon(n-1)]}\right\} \\ &= \frac{Var[s(n+1)+s(n-1)]}{Var[\varepsilon(n+1)+\varepsilon(n-1)]} = \frac{Var[s(n)]}{Var[\varepsilon(n)]} \end{aligned} \tag{7}$$

$$\begin{aligned} & SNR\left\{2A\sin\left(2\pi\frac{f}{fs}n+\theta\right)\cos\left(2\pi\frac{f}{fs}\right) + \varepsilon(n+1)+\varepsilon(n-1)\right\} \\ &= \frac{Var[2s(n)]}{Var[\varepsilon(n+1)+\varepsilon(n-1)]}\cos^2\left(2\pi\frac{f}{fs}\right) \\ &= 2\cos^2\left(2\pi\frac{f}{fs}\right)\frac{Var[s(n)]}{Var[\varepsilon(n)]} \end{aligned} \tag{8}$$

The factor $2\cos^2\left(2\pi\frac{f}{fs}\right)$ should be greater than 0.5 to make the SNR decrease less than 3 dB.

Hence, we can have

$$2\cos^2\left(2\pi\frac{f}{fs}\right) > \frac{1}{2} \Rightarrow 1+\cos\left(4\pi\frac{f}{fs}\right) > \frac{1}{2}. \tag{9}$$

The suitable range for frequency estimation is

$$0 < f < \frac{1}{6}fs, \tag{10}$$

which can be solved from Eq. (9).

In that case, the performance of traditional algorithm may degrade in the high frequency ($fs/6 < f < fs/2$) domain.

3. Fast adaptive iterative algorithm for precise frequency estimation with extend frequency range

In order to eliminate the above narrow band limitation and the constant noise interference, the differential equation is introduced from Eqs. (11)–(13).

$$\begin{aligned} y(n)+y(n-2) &= 2A\sin\left(2\pi\frac{f}{fs}(n-1)+\theta\right)\cos\left(2\pi\frac{f}{fs}\right) + \varepsilon(n)+\varepsilon(n-2) \\ &= 2s(n-1)\cos\left(2\pi\frac{f}{fs}\right) + \varepsilon(n)+\varepsilon(n-2) \end{aligned} \tag{11}$$

$$\begin{aligned} y(n-1)+y(n-3) &= 2A\sin\left(2\pi\frac{f}{fs}(n-2)+\theta\right)\cos\left(2\pi\frac{f}{fs}\right) + \varepsilon(n-1) \\ &\quad + \varepsilon(n-3) \\ &= 2s(n-2)\cos\left(2\pi\frac{f}{fs}\right) + \varepsilon(n-1)+\varepsilon(n-3) \end{aligned} \tag{12}$$