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Research on the piezoelectric torsional effect of a rectangular quartz disc and a novel drilling dynamometer

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ABSTRACT

The theoretical foundation of a new-type drilling dynamometer (i.e. the torsional effect of a rectangular piezoelectric quartz disc), the structure design and calibration of the drilling dynamometer are investigated in this paper. By using the theory of anisotropic elasticity and the Maxwell electromagnetism, the torsion stress and the distribution of surface charge densities of a rectangular quartz disc are calculated. According to the theoretical analyses of the bound charge densities, the detection electrodes are effectively disposed on the surface of the piezoelectric disc. The experimental results show that the torsional effect exists in the rectangular quartz disc and the bound charges are linear with the torque applied. Based on the torsional effect, a new type of drilling dynamometer is designed, in which only four quartz discs are used. The axial force measuring cell consists of two X0°-Cut quartz discs, and by the partitioned electrode method only two Y0°-Cut quartz discs can simultaneously sense the radial force and torque. After static calibrations, the torque sensor has full reached the dynamometer standard stipulated by CIRP-STCC. Undoubtedly the drilling dynamometer will provide a new method for measuring the drilling force and monitoring the drilling process.

1. Introduction

Since Curie brothers discovered piezoelectricity in quartz crystal in 1880, the piezoelectric properties of quartz crystal has been studied extensively, and the piezoelectric theory of quartz crystal based on stress analysis, namely, longitudinal effect, transverse effect and shear effect, has gradually established [1]. With the development of scientific technology and engineering requirements, the research on the bending effect, torsional effect and combined effect about quartz, etc. has been developed. Especially for the torsional effect, it will not only set up a new method for torque measurement, but also eventually establish the theoretical foundation of the piezoelectric theory of quartz crystal based on deformation analysis.

The document [2,3] investigated the torsional characteristics of a square quartz pillar. The approach was that the electrodes were disposed on the lateral surfaces of the pillar and a torque was applied on the end. Namely the surface acted on by the torque is normal to the surfaces disposed with electrodes. Because quartz crystal is so rigid and fragile, and the shear strength is small, that that geometry is difficult in fixation and installation. Therefore the research is only meaningful in theory and does not meet the requirement of torque measurement. Our previous research revealed that the torsional effect of a circular quartz disc existed, and used it to design a new type of torque sensor [4]. Since the circular quartz disc is difficult to cut from a quartz crystal and in the assembling process it is also inconvenient to orient, at present the majority of the quartz dynamometers use the rectangular quartz discs. Therefore further studies of the torsional effect of a rectangular disc are still essential.

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In this paper using the theory of anisotropic elasticity, the electromagnetic theory and piezoelectricity, the torsional effect of a rectangular quartz disc is investigated, and then directly using the torsional effect to measure the torque, a new type of drilling dynamometer is designed in which only four quartz discs are used.

2. Theoretical analysis of the torsional effect

The torsional effect refers to the production of bound electrical charges on the surface of a rectangular quartz disc by the imposition of mechanical torque, which is the theoretical foundation of the new-type drilling dynamometer.

2.1. Calculation of the torsion stress

Considering the elastic equilibrium of a rectangular quartz disc with the general rectilinear anisotropy, one of whose ends is fixed and the other is subjected to a twisting moment M_t, whose direction is along the z axis in Fig. 1. The dimensions of the quartz disc are the length $a = 21$ mm, inner radius $R = 6.5$ mm, and thickness $t = 1$ mm. In order to facilitate numerical calculation, we need rotate the crystallographic coordinate system $O'x'y'z'$ to the calculated coordinate system $Oxyz$. During a transformation of coordinate axis, the elastic compliance coefficients and the piezoelectric tensor change accordingly in terms of the transformation rule of tensors [5].

According to the theory of elasticity of an anisotropic body [6], the plane stress function $F(x, y)$ and torsion stress function $\varphi(x, y)$ must satisfy the differential equations (Eqs. (1) and (2)) and the boundary condition (Eqs. (3) and (4)).

$$\left.\begin{aligned} &L_4F + L_3\phi = 0 \\ &L_3F + L_2\phi = -2\vartheta + \frac{M_t}{2s_{33}}\left(\frac{s_{34}^2}{I_2} + \frac{s_{35}^2}{I_1}\right) \end{aligned}\right\} \quad (1)$$

where L_4, L_3 and L_2 are differential operators of the form

$$\left.\begin{aligned} &L_4 \equiv \beta_{22}\frac{\partial^4}{\partial x^4} - 2\beta_{26}\frac{\partial^4}{\partial x^3\partial y} + (2\beta_{12}+\beta_{66})\frac{\partial^4}{\partial x^2\partial y^2} - 2\beta_{16}\frac{\partial^4}{\partial x\partial y^3} + \beta_{11}\frac{\partial^4}{\partial y^4} \\ &L_3 \equiv -\beta_{24}\frac{\partial^3}{\partial x^3} + (\beta_{25}+\beta_{46})\frac{\partial^3}{\partial x^2\partial y} - (\beta_{14}+\beta_{56})\frac{\partial^3}{\partial x\partial y^2} + \beta_{15}\frac{\partial^3}{\partial y^3} \\ &L_2 \equiv \beta_{44}\frac{\partial^2}{\partial x^2} - 2\beta_{45}\frac{\partial^2}{\partial x\partial y} + \beta_{55}\frac{\partial^2}{\partial y^2} \end{aligned}\right\} \quad (2)$$

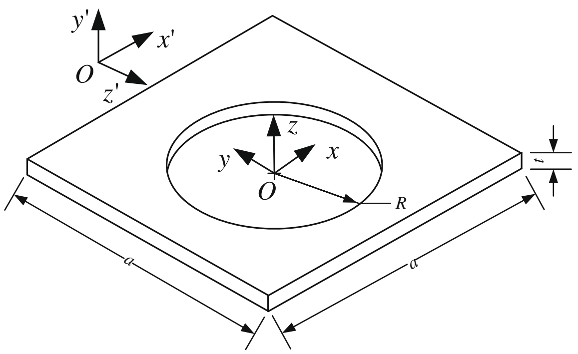

Fig. 1. Rectangular quartz disc applied by a torque.

$$\frac{\partial F}{\partial x} = 0, \quad \frac{\partial F}{\partial y} = 0, \quad \phi|_\Gamma = 0 \quad (3)$$

$$\left.\begin{aligned} &\int\int \tau_{xz}dxdy = 0, \quad \int\int \sigma_z ydxdy = 0 \\ &\int\int \tau_{yz}dxdy = 0, \quad \int\int \sigma_z xdxdy = 0 \\ &\int\int \sigma_z dxdy = 0, \quad \int\int(\tau_{yz}x - \tau_{xz}y)dxdy = M_t \end{aligned}\right\} \quad (4)$$

where I_1 and I_2 are the moments of inertia of the cross-sectional area with respect to the x and y axes respectively, s_{ij} are the elastic compliance coefficients, ϑ is the twisting angle per unit length, and M_t is a twisting moment. β_{ij} are the reduced elastic constants where the elements of β_{ij} are given by $\beta_{ij} = s_{ij} - \frac{s_{i3}s_{j3}}{s_{33}}\,(i,j = 1,2,4,5,6)$.

From the boundary condition Eq. (3), we can easily obtain the plain stress function:

$$F(x,y) = 0 \quad (5)$$

According to the theory of elasticity of an anisotropic body [6], the torsion stress function $\varphi(x, y)$ can be expressed in Fourier sine series:

$$\phi(x,y) = \frac{8a^2\left(\vartheta - \frac{M_t s_{34}^2}{4s_{33}I}\right)}{\beta_{44}\pi^3}\sum_{m=1,3,5,\ldots}^{\infty}\frac{1}{m^3}\left(1 - \frac{ch\frac{m\pi\mu}{a}y}{ch\frac{m\pi\mu}{2}}\right)\sin\frac{m\pi(2x+a)}{2a} \quad (6)$$

where μ is a coefficient whose value is $\sqrt{\beta_{44}/\beta_{55}}$.

Because the region of the quartz disc's cross section is multiply connected, suppose that the torsion stress is zero on the outer contour of the rectangle disc, the torsion stress on the inner contour should be a constant k:

$$k = \phi(x,y)|_{x^2+y^2=R^2} = \frac{8a^2\left(\vartheta - \frac{M_t s_{34}^3}{4s_{33}I}\right)}{\beta_{44}\pi^3} \times \sum_{m=1,3,5,\ldots}^{\infty}\frac{1}{m^3}\left(1 - \frac{ch\frac{m\pi\mu}{a}}{ch\frac{m\pi\mu}{2}}\right)\sin\frac{m\pi(2x+a)}{2a} \quad (7)$$

The area A enclosed by the inner contour is:

$$A = \pi R^2 \quad (8)$$

By means of the boundary condition $M_t = 2\int\int \phi dxdy + 2kA$ [7], considering Eqs. (6)–(8) we can calculate the twisting angle per unit length:

$$\vartheta = 6.4223 \times 10^{-4} M_t \quad (9)$$

Substituting Eq. (9) into Eq. (6), the final expression for the torsion stress function can be rewritten as:

$$\phi(x,y) = 3.7946 \times 10^3 M_t \times \sum_{m=1,3,5,\ldots}^{\infty}\frac{1}{m^3}\left(1 - \frac{ch2.5my/a}{ch1.25m}\right)\sin\frac{m\pi(2x+a)}{2a} \quad (10)$$

Using the definitions of the plane stress function $F(x, y)$ and the torsion stress function $\varphi(x, y)$, the components of state of stress can be calculated as follows:

$$\sigma_x = \frac{\partial^2 F(x,y)}{\partial y^2} = 0, \quad \sigma_y = \frac{\partial^2 F(x,y)}{\partial x^2} = 0, \quad \tau_{xy} = -\frac{\partial^2 F(x,y)}{\partial x\partial y} = 0 \quad (11)$$