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Analytical design of dual-tone signal for ADC phase-plane testing

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ABSTRACT

An analytical description of the phase-plane behavior of a dual-tone signal for nonlinear analog-to-digital converters testing is presented. A quality index for the phase-plane coverage of the dual-tone test signal is proposed and evaluated. Conditions for optimal input frequency selection are analytically derived for practical calibration applications, and numerical results highlighting the working of the proposed approach are provided.

1. Introduction

Current technical enhancement trends make metrological characterization of analog-to-digital converters (ADCs) a stimulating scientific challenge [1,2]. In this scenario, a pre-eminent role has been played by phase-plane error modeling [3–10]. The ADC dynamic behavior is described as a function of the output code as well as the instantaneous slope of the input, i.e. in the "phase plane". Various solutions, either aiming at (i) maximizing generality and discriminating ADC behavior for a larger class of input signals [3–6], independently of the ADC architecture or (ii) maximizing effectiveness and exploiting an analytical a-priori approach for most popular ADC architectures [7,8], making use of their peculiarities, were proposed. Phase-plane modeling was improved mainly in model identification, namely, in the experimental burden [5,6,10], and, more recently, in calibrating signals [9–12]. Last developments were related to the introduction of a dual-tone test signal [11–15] owing to its intrinsic capability of mapping the phase-plane extensively. Even though the impact of sampling effects has been investigated by Blair [12], theoretical conditions for best phase-plane coverage have not been analytically defined yet.

In this paper, a comprehensive analytical approach to the usage of the dual-tone as a test signal for phase-plane-based metrological characterization of ADCs is presented. In Sections 2 and 3, basic definitions and analytical description of phase-plane behavior of the dual-tone signal are provided. In Section 4, a metrological quality index, assessing the phase-plane coverage, is proposed and evaluated. On this basis, in Section 5, the impact of sampling on the coverage is assessed. Finally, in Section 6, an operating guide for optimal input frequency selection in practical testing applications is provided and in Section 7, numerical results that corroborate the proposed approach are presented.

2. Basic definitions

Let the dual-tone signal x_{DT} and its normalized derivative s_{DT} be expressed by

$$x_{DT}(t) = 1/2 \cdot \cos(\omega_A \cdot t) + 1/2 \cdot \cos(\omega_B \cdot t), \tag{1}$$

where ω_A is approximately equal to ω_B, with $\omega_A < \omega_B$ [11,12], and the amplitude is normalized so that $x_{DT} \in [-1; 1]$, and

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$$s_{DT}(t) = \frac{2}{\omega_A + \omega_B}\frac{dx_{DT}}{dt}(t) = -\frac{\omega_A}{\omega_A + \omega_B}\sin(\omega_A \cdot t) - \frac{\omega_B}{\omega_A + \omega_B}\sin(\omega_B \cdot t), \quad (2)$$

where the amplitude is also normalized so that $s_{DT} \in [-1; 1]$.

In practical applications, the dual-tone usage is made easier if additional conditions, related to the choice of the two input frequencies, are met.

Let the following quantities be defined as:

$\omega_S = 2\pi \cdot f_S$ angular sampling frequency,
$N_S = 2^{ns}$ number of samples per record ($ns \in \mathbb{N}$), and
$\omega_1 = \omega_S/N_S$ angular frequency resolution, i.e. angular frequency of the first bin, of the FFT spectrum.

Spectral leakage is avoided if the angular frequencies ω_A and ω_B are integer multiples of ω_1, i.e.,

$$\omega_A = N_A \cdot \omega_1 \quad \omega_B = N_B \cdot \omega_1, \quad (3)$$

with N_A and N_B integers, and $0 < N_A < N_B$, whereas, for sampling theorem,

$$\omega_B < \omega_S/2 \iff N_B < N_S/2. \quad (4)$$

From (3), it follows that x_{DT} is periodic, with angular frequency

$$\omega_{DT} = N_{DT} \cdot \omega_1 \quad N_{DT} = GCD(N_A, N_B) \quad 1 \leqslant N_{DT} \leqslant N_A, \quad (5)$$

where GCD stands for the greatest common divisor.

Coherent sampling of the input signal is assured if [16]

$$\mathrm{GCD}(N_{DT}, N_S = 2^{ns}) = 1 \iff N_{DT}\mathrm{odd} \iff N_A\mathrm{odd} \vee N_B\mathrm{odd}. \quad (6)$$

A signal on the phase plane is represented by the plot of its instantaneous derivative against its instantaneous value. Thus, the head of the vector ($x_{DT}(t)$; $s_{DT}(t)$) provides a representation of the dual tone x_{DT} falling into the normalized squared area $[-1; 1] \times [-1; 1]$. Moreover, as the two tones are sinusoidal, the coverage area will be contained in a circumference centered at the phase plane origin (0; 0). Therefore, the area under analysis for coverage purposes will be the unitary circle.

3. Phase-plane behavior of the dual tone

In the following, (i) an intuitive and (ii) a more formal approach to the behavior of the dual tone in the phase plane are described.

3.1. Intuitive approach

Let the single-tone signal x_{ST} and its normalized derivative s_{ST} be considered:

$$x_{ST}(t) = A_{ST}\cos(\omega_{ST} \cdot t),$$
$$s_{ST}(t) = \frac{1}{\omega_{ST}}\frac{dx_{ST}}{dt} = -A_{ST}\sin(\omega_{ST} \cdot t). \quad (7)$$

The trace of x_{DT} on the phase plane is a circumference centered at the origin, with radius R_{ST}, equal to A_{ST} (Fig. 1a):

$$R_{ST}^2 = x_{ST}^2 + s_{ST}^2 = A_{ST}^2. \quad (8)$$

This circumference, as any resulting path on the phase plane, must be drawn clockwise over the time: (i) when the signal amplitude increases, the trace is drawn from left to right. As the derivative is positive, the trace must be crossing the first or second quadrants of the phase plane and (ii) when the signal amplitude decreases, the trace is drawn from right to left. As the derivative is negative, the trace must be crossing the third or fourth quadrants of the phase plane.

The time evolution of the dual tone (1) can be expressed as the result of a 100% amplitude modulation:

$$x_{DT}(t) = A_D(t) \cdot x_C(t) = \cos(\omega_D \cdot t) \cdot \cos(\omega_C \cdot t),$$
$$\omega_D = \frac{\omega_B - \omega_A}{2} = \frac{N_B - N_A}{2}\omega_1 \quad \omega_C = \frac{\omega_B + \omega_A}{2} = \frac{N_B + N_A}{2}\omega_1. \quad (9)$$

In this way, $x_{DT}(t)$ can be interpreted as the succession of values assumed by the simple sine wave $x_C(t)$, the carrier function, when multiplied by a slowly variable amplitude, $A_D(t)$, the modulating function. During the first quarter of period of A_D, $[0; T_D/4[$, the amplitude A_D decreases monotonously from 1 to 0. Hence, during this interval, at each full period T_C of the carrier, the curve described on the phase-plane is not locked within itself, as for a simple sine wave, but it tightens as it curls (Fig. 1b). During successive periods of the carrier, the curve will curl and tighten itself further, resembling a spiral, reaching its center at $t = T_D/4$, when $A_D(T_D/4) = 0$ (Fig. 1c, solid line).

Analogously, during the following quarter of period $[T_D/4; T_D/2[$, as the absolute value of A_D increases from 0 to 1, a new spiral will be traced from the center to the periphery, with increasing amplitude (Fig. 1c, dashed line). Notice that, due to the clockwise movement, this expanding spiral necessarily follows a different path from the previous one, because following back through the same path would imply an anti-clockwise rotation. Eventually, alternate contracting and expanding spirals will be drawn at each quarter of period interval

$$\Delta T_{DJ} = [J \cdot T_D/4; (J+1) \cdot T_D/4[, \quad (10)$$

where J is a non-negative integer. Furthermore, as it will be demonstrated below, within a full period T_{DT} of the dual tone, all the spirals traced follow different paths.

3.2. Analytical approach

In the following, the phase-plane behavior of x_{DT} will be described analytically trough the following steps:

Step 1: Express the squared distance R_DT^2 between the trace (x_{DT}; s_{DT}) and the origin (0; 0) as a function of time, and restrict the analysis to the time interval $\Delta T_{D0} = [0; T_D/4[$.
Step 2: Divide the phase plane in origin-centered adjacent annuluses (circular crowns) covering the whole unitary circle (Fig. 2).
Step 3: Demonstrate that each annulus contains one full turn of the trace (x_{DT}; s_{DT}), thus implying a spiroidal movement over the time.