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A quasi-static FEM for estimating gear load capacity

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ABSTRACT

Measurement of time-varying load capacity of gears is a vital process prior to manufacturing. There are two ways to evaluate the strength of gears, the analytical method proposed in AGMA and ISO standards and the finite element method (FEM). However, owing to the complex geometries and the use of empirical values, analytical model is not precise to perform the exact calculation. The current dynamic FEM and static FEM also have some disadvantages may cause the analysis of time-vary load capacity difficult. Therefore, the first aim of this study is to develop a new quasi-static FEM based on ANSYS Workbench to conduct the analysis of time-varying load capacity of gear system. The second aim is to compare results of the traditional method based on AGMA equations and quasi-static FEM. The effects of tip relief are also analysed using this new method.

1. Introduction

Gear drive is one of the fundamental components of rotating machines and is widely used in modern industry due to the properties of compactness and high torque-to-weight ratios [1]. As a periodic function caused by the change in the number of contact tooth pairs and the contact positions of the gear teeth, the contact stress on teeth surface and the bending stress at gear root vary periodically with time. They are two failure modes that are important causes of gear tooth failures. Contact stress leads to pitting on gear surface and bending stress lead to tooth breakage at gear root. Thus it is vital to understand the time-varying load capacity of gears.

The strength of gears is often roughly estimated using the AGMA [2,3] and ISO [4–6] gear rating standards prior to manufacturing. However, these standards are not precise to perform the exact calculations. This is mainly due to the complex geometries which make analytical modelling of the gears to determine its load capacity difficult. Furthermore, the introduction of some empirical values for rating factors decreases the computation accuracy of these two standards. Therefore, a growing number of researchers start to examine the gear load capacity with FEM.

Hwang et al. [7] used two-dimensional static FEM to analyse the contact stress of a pair of mating gears. Compared with the AGMA standard, they found the results obtained by FEM were more severe than that of the AGMA standard. Barbieri et al. [8] presented a adaptive grid-size finite element model to perform the Loaded Tooth Contact Analysis (LTCA). Its computation accuracy was improved compared with previous methods. Patil et al. [9] studied the contact stresses among the helical gear pairs, under static conditions, by using a 3D finite element method. Gear sets with different helical angle were analysed. They found that the effect of friction was varied at the point of contact. Using the finite element dynamics method, the contact stresses of spur gears were also investigated by Qin and Guan [10]. The stresses near the engagement and recess areas are also found to be greater than the static contact conditions and thus result in low fatigue life, particularly at high speeds. Wu et al. [11] implemented a combined 3D face contact and FEM to investigate the contact stress of two spur gears. Two spur gear teeth were

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conducted in 11 different contact positions during a full mating process. Their method provided a complete and effective solution to contact problem in a quasi-dynamic manner. An integrated finite element analysis was also conducted in Ref. [12]. The relationship between the pressure angle the contact stress in the gear pair system was presented. The results were expected to enhance the technology of gear system design.

Costopoulos and Spitas [13] investigated the gear fillet stresses by using FEM. They found that the load capacity of asymmetric gear could be increased up to 28% compared to the standard 20° involute teeth. Based on FEM method, Kramberger et al. [14] developed a strain-life method to calculate the root stress of gear teeth. This model could be used to determine the complete service life of spur gear. Li [15] investigated the effect of addendum on tooth bending strength in 2008 by using FEM. He found that the increment of addendum can increase number of contact teeth, then this increment can reduce equivalent bending stress at gear root. The effects of machining errors, assembly errors and tooth modifications on bending strength were also analysed by Li with similar FEM [16,17].

In the above-mentioned literatures, there are two ways to evaluate the gear load capacity, the dynamic FEM and the static FEM. The bigger disadvantage of dynamic FEM is time-consuming. If dynamic FEM is used to numerically estimate the load capacity of meshing gear pairs, the integration time step needs to be smaller. The dynamic FEM is also sensitive to the normal contact stiffness, penetration tolerance and mesh density. The gear contact may cause convergence difficulty due to its non-linear characteristics.

The static FEM also has drawbacks because this method was based on single point estimation of the load capacity. In this case, for time-vary load capacity measurement, the finite element analysis needs to be repeated at every contact point of gear pairs. Therefore, in order to avoid the repeated setup process and save computation time, scholars developed various types of programme codes to analyse the time-vary load capacity. It is however difficult for an inexperienced researcher without sufficient ability in the use of computer language.

In this study, instead of focusing on the complex programming codes, the contact stress and bending stress corresponding to the rotation position of gear pair is investigated using a quasi-static FEM. Our attempt is to develop a new FEM based on ANSYS Workbench to conduct the analysis of time-varying load capacity of gear system. Furthermore, another aim of this study is to compare results of the traditional method based on AGMA equations and quasi-static FEM. The effects of tip relief are also analysed using this new method.

2. Calculation of gear load capacity

To estimate the load capacity of spur and helical involute gear teeth, ISO Standard 6336 and AGMA Standard 2101-C95 established a common base for rating various types of gears for differing application. The analytical method presented in AGMA standard will be used to compare with the proposed finite element method.

2.1. Contact stress calculation

Based on the Hertzian theory, the contact stress equation of AGMA standard is given as:

$$\sigma_H = Z_E\sqrt{F_t K_o K_v K_s \frac{K_H}{2r_p W Z_I}}. \tag{1}$$

Z_E is the elastic coefficient can be written as:

$$Z_E = \left[\frac{1}{\pi\left(\frac{1-\upsilon_P^2}{E_p}+\frac{1-\upsilon_G^2}{E_G}\right)}\right]^{1/2} \tag{2}$$

where υ and E are the Poisson's ratio and Young's modulus, respectively. Suffix P represents the pinion and G stands for gear as shown in Fig. 1. F_t is the transmitted tangential load applied on gear teeth. K_o is the overload factor. K_v is the dynamic factor. K_s is the size factor. K_H is the load distribution factor. r_p is the pitch radius of pinion and W is the face width.

In this study, emphasis is given to calculate the contact stress of external involute spur gears. Therefore, the geometry factor Z_I can be calculated as:

$$Z_I = \frac{\cos\phi_t \sin\phi_t}{2m_N}\,\frac{m_r}{m_r+1} \tag{3}$$

where ϕ_t is the transverse pressure angle. m_N is the load-sharing ratio ($m_N = 1$ for spur gears). m_r is defined as the gear ratio.

$$m_r = \frac{Z_G}{Z_P} \tag{4}$$

where Z_G and Z_P are the teeth number of gear and pinion, respectively.

2.2. Bending stress of gear root

Lewis equation is derived from the basic beam bending stress equation, which forms the basis of the AGMA bending stress equation used nowadays. The final equation of bending stress, considering the same empirical factors

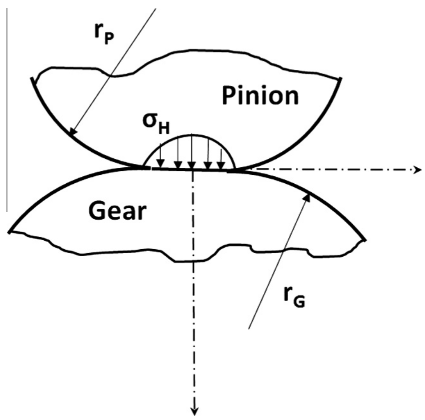

Fig. 1. Schematic of Hertzian contact.