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The minimum zone evaluation for elliptical profile error based on the geometry optimal approximation algorithm

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ABSTRACT

To realize a more accurate evaluation for elliptical profile error, based on the minimum zone geometry optimization approach method, an algorithm for evaluating elliptical profile error is presented. Firstly, based on the least squares method, a reference ellipse and a pair of initial reference points are determined, taking the pair of reference points as reference points, auxiliary focuses is established and auxiliary ellipses are constructed by using the auxiliary focuses. Secondly, the elliptical profile errors are calculated, when the reference ellipse and the auxiliary ellipses as supposed ideal ellipses. Through comparing, judging and changing the reference points, constructing new auxiliary points and auxiliary ellipses, forming new assumed ideal ellipses, the minimum zone evaluation for elliptical profile error is realized finally by using the iterative approximation method. The process and the steps of solving elliptical profile error are described in detail, the mathematical formulas are given. Which shows this algorithm can not only get the results accurately but also stably.

1. Introduction

The elliptical profile has many practical and peculiar applications in engineering, such as the spectrophotometer, piston-section of IC engines and the elliptical gear. In the spectrophotometer, the generatrix shape error of the ellipsoidal mirror is directly affecting its optical performance; the pitch contour error of the elliptical gear affects transmission performance and the profile error of piston skirt is a serious constraint to ensure good thermal effect of the IC engines. So its necessary to study the elliptical profile error evaluation algorithm, which is of great significant to ensure the machining quality of the elliptical profile parts.

Elliptical profile tolerance zone is an area between two isometric lines of ellipse equidistant from ideal ellipse, in today's international standards, the straightness error, flatness error, roundness error and cylindrical error discriminate methods have been listed, but the elliptical profile error discriminate method is still not listed.

Until now, a clear definition of elliptical profile error as well as the evaluation algorithm for elliptical profile error has not been given by international standards. Murthy [1] proposed three different methods for the evaluation of the elliptical profiles: based on normal least squares fit, bivariate Gaussian distribution and general second degree equation. In the 3rd method, the deviation from the property of the ellipse is minimum. Fitzgibbon et al. [2] applied the least squares fitting, under the normalization $4ac - b^2 = 1$, to minimize the sum of squared algebraic distances from the points to the ellipse. Based on the principle of least method and using "variable substitution" technique, Liu et al. [3] described an algorithm to evaluate the form error of profile. Compared with classical simple and iterative methods, when the square sums of the distances to the given points is minimal, Gander et al.

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[4] presented several algorithms to compute the elliptical profile error. Based on a least-square orthogonal distance, Ahn et al. [5] developed the fitting algorithms of ellipse. All the necessary information for iterative orthogonal distances fitting can be obtained from the derivatives of the coordinate transform. Wijewickrema et al. [6] discussed a direct calculation of the orthogonal distance, which requires less accurate initializations, uses simpler calculations and produces more accurate results. Hou et al. [7] given the unified description of parameter vector function for designed curve of complicated plane profiles, and derived the distance function from measured points to the design curve by the theory of differential geometry. Cui et al. [8] proposed an unbiased minimum variance estimator to estimate the parameters of an ellipse. Based on least square and under the parameter constraints, Zou and Bo [9] studied the fragmental ellipse fitting algorithm on the position and shape of the ellipse. Based on the mathematical equation of ellipse, Li and Shi [10] presented a data processing method of ellipse profile. Zhang et al. [11] proposed the method with coordinate system being adjustable automatically in the auto-spotting evaluation of profile of freeform curve. The auto-adjustment between the measured profile and theoretical profile can be achieved by applying the methods of corresponding characteristic points and DFP-one dimension search.

From the above references, it can be seen the simplified algorithms might not bring about accurate evaluations, and the optimization algorithms present difficulties in deciding initial iteration points and step lengths of optimization. Furthermore, the computation process might be complex resulted by non-linear equations.

Combined with the geometric characteristics and evaluation problems of the elliptical profile error, a evaluation method of the minimum zone evaluation for elliptical profile error (MZE) based on the Geometry Optimization Approximation Algorithm (GOAA) is proposed in this paper, in which the elliptical profile error can be evaluated effectively and accurately.

2. Evaluation principle of GOAA

Firstly, a reference ellipse is determined by the least square method and the least-square elliptical profile error (written as e_0) is calculated. Then when two focuses of the reference ellipse are set as initial reference points, two squares are set respectively and eight auxiliary points (the vertex of the square) can be obtained. When eight auxiliary points are set as the focuses of the ellipses (Fig. 1), the 16 auxiliary ellipses can be constructed. Assuming auxiliary ellipse as the ideal ellipse of the measured ellipse, when the range values of distance of all measurement points to the auxiliary ellipses are calculated, the 16 elliptical profile errors can be obtained, where the minimum is written as e_1. If $e_0 \leqslant e_1$, keeping the reference points unchanged, reconstructing a new square with the side length scaling down (like the bisection method, the golden section method), and obtaining the new auxiliary points, e_1 would be solved again. If $e_0 > e_1$, setting the corresponding focuses of e_1 of the assumed ideal ellipses as the new reference points, reconstructing a new square can be constructed with the side length unchanged, new auxiliary points can be obtained. Repeating the above process, when the side length of the square created is less than a small amount set, it can be considered that the assumed ideal ellipse searched is already very close to the actual ideal ellipse, when the search terminate, the minimum of e_0 and e_1 is obtained as the minimum zone the elliptical profile error of the measured ellipse.

3. Evaluation steps of GOAA

3.1. Determination of the initial parameters

The coordinate values of the measurement points on the elliptical contour are set as $p_i(x_i, y_i)(i = 1, 2, \ldots, N)$, the initial reference points are the least squares ellipse focus. e_0 equals to the value of the square side length (f)

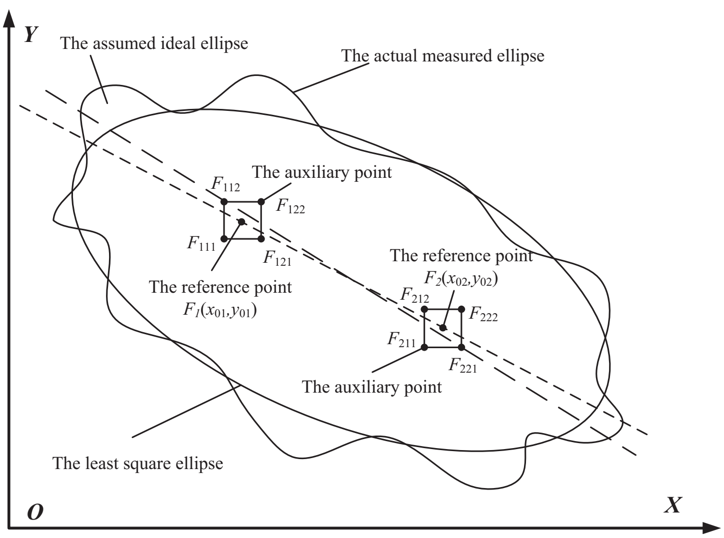

Fig. 1. The principle of the GOAA.