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Bounds on the performance of analog-to-digital converter look-up table post-correction

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ABSTRACT

Analog-to-digital converter additive post-correction using look-up tables is considered. The problem of successfully predicting the converter's performance after correction is treated in particular. An accurate expression is provided that predicts the ADC performance after correction. The expression depends on differential non-linearity, random noise variance, and the numerical precision of the correction terms. The theory shows good agreement when compared with simulations and experimental converter data. The results are useful when designing systems involving ADCs and post-correction, since the performance parameters can be obtained with knowledge of a few ADC intrinsic parameters and the correction system resolution.

1. Introduction

Analog-to-digital converter (ADC) post-correction has been proposed in many different forms, many of these applying look-up tables (LUTs) [1]. The most straightforward form of LUT correction is the static LUT, applied for instance in [2–5]. In this form, a single output sample from the ADC is used as an address for a digital memory, in which either correction terms or corrected values are stored. The name is static since it produces the same correction regardless of the dynamic properties of the signal, such as frequency, slope and history. Several methods for introducing signal dynamics in the LUT addressing have been proposed. State-space methods, using current and past samples in conjunction to form the address have been proposed, e.g., in [6–11]. Phase-plane method, where the current sample and (an estimate of) the input signal slope is used to form the table address, are applied, e.g., in [12–16]. A combination of the two is also suggested in [17]. The work in the present paper is oriented towards static look-up tables. The results are nevertheless useful also for designing dynamic correction systems. A successful deployment of dynamic compensation should outperform a static correction scheme, wherefore the results presented below could be used as pessimistic approximations for dynamic correction performance.

When designing ADC post-correction systems, it is of great interest to predict the performance that can be achieved. The performance will, naturally, depend on the characteristics of the ADC at hand. Furthermore, in a practical post-correction application it is very likely that the correction values will be stored with fixed-point precision, which of course affects the corrected output. However, most of the evaluations and experiments reported in the literature have been conducted with infinite precision in the representation of the correction values stored in the LUT. One of few exceptions is [18], where experimental results indicated that the precision of the correction values strongly affect the outcome of the correction. In this paper, we will present a theory linking the ADC performance after post-correction with a number of design parameters. In particular, an expression for the signal-to-noise and distor-

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tion ratio will be provided. We will show that the performance of the corrected ADC, in terms of signal-to-noise and distortion ratio, can be accurately predicted with knowledge of a few parameters, viz. the variance of the dynamic non-linearity sequence, the input noise variance, the ADC resolution, and the correction value precision. The theory is verified by computer simulations as well as experiments using a state-of-the-art ADC.

The remainder of the paper is organized as follows. First, the quantization and correction model is introduced in Section 2, along with definition and calculation of mean squared error. Then, the effects of the ADC non-linearities are analyzed in Section 3. In Section 4, the impact of limited precision correction values is quantified, followed by a short note on input noise in Section 5. The results are combined in Section 6 to a unified equation predicting the ADC performance after correction, taking all the above aspects into account; this is one of the major contributions of the present paper.

The results are verified by simulations in Section 7, and also using experimental ADC data in Section 8. The outcome of the experiments are discussed in the concluding Sections 9 and 10.

2. Data converter and correction system model

In this section, the ADC model is introduced, along with the correction scheme and the theories for optimal correction. The mean square error for the optimally corrected ADC is calculated.

2.1. ADC model

Consider an ADC with continuous-time input $s(t)$ and discrete-time output $x(n)$, as depicted in Fig. 1. The input is a continuous-time signal, which is sampled to $s(n)$ at the sampling instants $t = n/f_s$, where n is the integer time index. The ADC is assumed to possess an ideal sample-and-hold circuit. Thus, the sampling is excluded from the forthcoming analysis, and the sampled signal $s(n)$ is regarded as input to the system; the terms 'ADC' and 'quantizer' will be used interchangingly to denote this ADC model with ideal sampling. The quantizer has b bits, resulting in $M = 2^b$ quantization levels. The quantizer output, denoted $x(n)$, is a quantized version of $s(n)$. The quantization operation is defined by M disjunct regions $\mathscr{S}_0$ through $\mathscr{S}_{M-1}$, which together covers the entire input range. Each quantization region $\mathscr{S}_i$ is associated with one output level x_i which is assumed to be fixed but otherwise arbitrary. The quantization is defined such that $x(n) = x_i$ if $s(n) \in \mathscr{S}_i$. Also, the width of a quantization region is denoted the code bin width. The notation $x(n) = Q(s(n))$ is used to denote the quantization operation. Note that $Q(\cdot)$ does not necessarily have to be a uniform quantization, but represents the actual transfer function of the ADC at hand, as defined by the quantization regions. The quantizer is also assumed to be non-dynamic, i.e., the output of the quantizer at time n depends only on the input at the same instant.

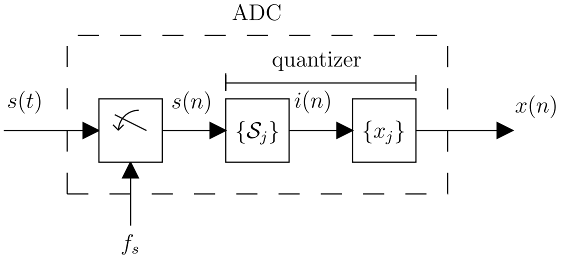

Fig. 1. A model for the ADC converter with the quantization represented as a two-step operation.

Throughout the analysis, the input value $s(n)$ at time index n is modeled as drawn from a stochastic variable S with probability density function (PDF) $f_S(s)$. The temporal properties for S are immaterial since the quantizer is assumed to be non-dynamic. As a natural consequence, the output is also stochastic, although the mapping from quantizer input to output by means of the quantization regions $\{\mathscr{S}_j\}$ is deterministic.

2.2. Post-correction system

A static additive correction as described for instance in [2] is connected to the quantizer output. Fig. 2 depicts the correction system. The corrected value y is produced by adding a correction term $e(x)$ to the output x so that $y = x + e(x)$. Every possible output value $x \in \{x_j\}_{j=0}^{M-1}$ is associated with a correction term $e(x) \in \{e_j\}_{j=0}^{M-1}$. The correction is static in the sense that the correction value produced at time index n depends on the quantizer output at time index n only, and not on past or future output values. Any memoryless function mapping $x \rightarrow y$ can be represented in this fashion.

Optimal correction values in the sense of minimizing the mean square error $\mathrm{E}[(S-y)^2]$ are used (note that y, like x, is a function of S and that the expectation is with respect to S). In [19] the minimum-mean-squared-error (MMSE) optimal correction values were derived. The result – reiterated here for convenience – is that if the input is drawn from a random variable with PDF $f_S(s)$ and the quantization regions $\{\mathscr{S}_j\}$ are assumed fixed, the optimal correction values are given by

$$e_{j,\mathrm{opt}} = \arg\min_y \mathrm{E}[(y-S)^2 | S \in \mathscr{S}_j] = \frac{\int_{s\in\mathscr{S}_j} s f_S(s)\mathrm{d}s}{\int_{s\in\mathscr{S}_j} f_S(s)\mathrm{d}s} - x_j. \quad (1)$$

This is the correction that would be used if the correction values could be represented using infinite precision. Note also that if S is uniform (at least within each quantization region), $f_S(s)$ is (piecewise) constant and we obtain "midpoint correction" – the corrected value $x_i + e_i$ is the midpoint of the corresponding quantization region $\mathscr{S}_i$.

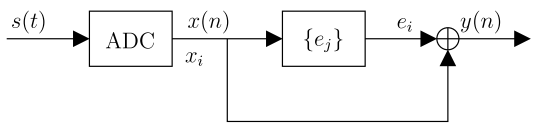

Fig. 2. Additive correction system.