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Shaping the trajectory of the billiard ball with approximations of the resultant contact forces

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ABSTRACT

This paper presents mathematical models of the contact pressure distribution on a circular contact area and the corresponding rolling resistance. Hertzian pressure distribution is distorted in a special way in order to move rolling centre outside the geometrical centre of the contact area. With the assumption of fully developed sliding and classical Coulomb friction law on each element of the contact, integral models of the total friction force and moment reduced to the contact centre are given. In order to improve the convenience of use of the contact models in numerical simulations of rigid body dynamics and decrease their computational cost, special approximations of the integral models of friction force and moment are proposed. Moreover, special modifications of the corresponding expressions for friction forces and rolling resistance are proposed, which allows avoiding their singularities for vanishing relative motion of the contacting bodies. The application of the proposed contact models in mathematical modelling of a rigid ball rolling and sliding over a deformable table is presented. Furthermore, possibilities of use of the developed simulation models in shaping the billiard ball's trajectory are presented.

1. Introduction

Classical understanding of friction model is a relation between single component of friction force and one-dimensional relative displacement of the contacting bodies. This relation can possess different levels of complexity, beginning from the classical Coulomb law, ending on more advanced relations taking into account other properties of friction, including also additional state variables. These kinds of models are applied directly in mathematical descriptions and investigations of dynamical systems with frictional contacts, where at each point of the contact area the same relative motion of the contacting bodies takes place. Therefore, they can be also applied in the modelling of frictional contacts of bodies in three-dimensional space, if the contact can be considered as a point contact or plane and non-deformable without relative rotation of the contacting bodies in the contact plane.

However it occurs, that in the daily life one can encounter many examples of mechanical systems in three-dimensional space, where the assumption of the same relative motion of the contacting bodies at each point of the contact area is not fulfilled. One can enumerate here such examples like dynamics of rolling bearing, billiard ball, Thompson top, Celtic stone, wheel-soil or wheel-rail of the vehicles, and many others issues encountered in robotics. The local deformations of the bodies can be small enough with comparison to their dimensions, so the global motion of the bodies can be treated as rigid body motion. However, the shape and size of the contact zone can influence the global dynamics of the body. The full solution to this problem can be achieved with the use of space discretization and then one of such numerical methods like finite element method or, under special assumptions, the integration over the contact area. The last case corresponds to the assumptions used in the present paper. The space discretization leads however to significant increase of computational time. From the point of view of realistic and fast simulations of certain class of mechanical systems, it is important to construct special approximate models of the resultant contact forces.

Contensou in [1] indicated that moment of friction and relative angular velocity around the axis perpendicular to the contact plane can be very important in dynamics of some mechanical systems, i.e. quickly spinning tops. He presented analytical form (in terms of elliptic integrals) of dependence of the friction force on the relative sliding linear and angular velocities, assuming the classical Coulomb friction law and fully developed sliding on a circular contact area, with Hertzian contact pressure distribution. Zhuravlev in [2] showed that for a parabolic contact pressure distribution and a special choice of the co-ordinate system, one can find expressions for friction force and moment including only elementary functions.

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Since these expressions are rather complicated and inconvenient in use for modelling of real systems and numerical simulations, he proposed special approximations of the exact expressions based on the Padé approximants as well. More results concerning the use of higher order Padé approximations in modelling friction force and moment are presented in [3]. Model of rolling resistance with the related special contact pressure distribution on circular contact area and the corresponding Padé approximations of the resultant friction force and moment were presented in [4]. Furthermore, the new families of models and approximations were proposed in [5], which can be understood as generalizations of the previously mentioned approaches. In particular, the model of contact pressure distribution and rolling resistance was extended to the case of elliptical contact area. Moreover, the approximations of the friction force and moment were generalized and their ability to fit the integral models or experimental data was increased.

The above mentioned integral models and their approximations concern the case of fully developed sliding. However, during simulations of mechanical systems one can manage not only with fully developed sliding, but also with stick mode, as well as transition between them. In general, there are three different approaches to this problem: time-stepping methods, event-driven methods and regularization methods. Kudra and Awrejcewicz in [6] presented numerical scheme and the corresponding examples of simulation of stick-slip oscillations, where both the linear and rotational relative motion of the contacting bodies takes place. The numerical algorithms were constructed as an event-driven scheme, where the approximations of the integral friction model for both the fully developed sliding and stick mode (determination of the end of the stick mode) were used. In [7] the corresponding regularized approximations of the integral models of friction for fully developed sliding were presented, where the singularities (for vanishing relative motion) were removed. It was shown that the developed models of friction lead to the same results as event-driven algorithm presented in [6].

The application of the special approximate model of the resultant contact forces in the billiard ball rolling and sliding on the deformable table is considered in this paper. Time consumption of the numerical calculation is crucial in simulation of the billiard game in which the models of resultant contact forces are expected to be computationally effective. Shaping the billiard ball's trajectory with the consideration of the influence of shape and size of the contact zone on the dynamics of the ball has not been found in the state of the art. Most of the found applications use the pre-prepared engines or large libraries without going into constituent parts. As it is written in [8], Open Dynamics Engine (ODE) is used. It is an open source physics library, which allows providing a realistic environment when 3D objects are colliding. ODE is used to simulate collisions between objects of different shapes. Another techniques and research to develop a game with reality is virtual reality technology with Visual, Auditory and Haptic Sensation presented in [9]. In [10] the simulation of the billiard game is based on the marker detection. However, none of them mentioned about the model of contact forces considering the shape and size of the contact zone and shaping the ball's trajectory. They focused on problem of collisions and communications between human and computer.

Section 2presents mathematical models of the contact pressure distribution, rolling resistance (2.1), integral model of friction force and moment (2.2) for fully developed sliding on a circular contact area, as well as their special approximations (2.3) and regularizations (2.4). Section 3 exhibits a mathematical model of a rigid billiard ball rolling and sliding over a deformable table (3.1). Additionally, this section presents the abilities of the developed models to predict and shape the billiard ball's trajectory. Section 4 contains some concluding remarks.

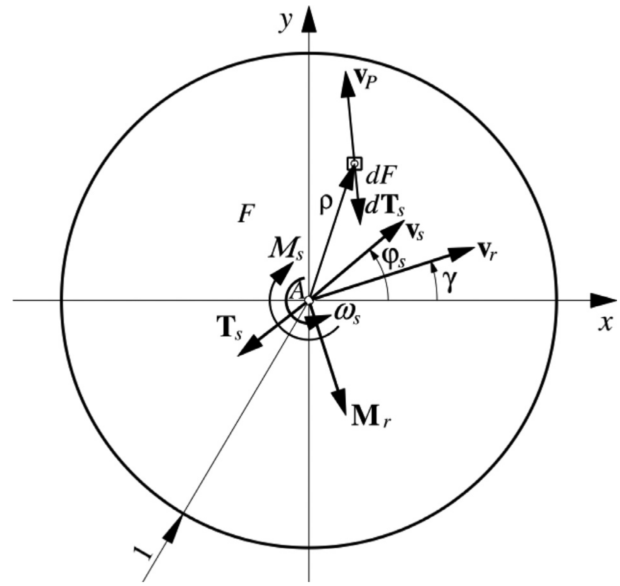

Fig. 1. The circular contact area with the characteristic relative velocities and the resultant forces and force couples acting on the body lying above the contact patch.

2. Modelling of the contact forces

2.1. Contact pressure distribution and rolling resistance

Let us consider a dimensionless circular contact area F presented in Fig. 1, with the Cartesian coordinate system $Axyz$ with axes xy lying in the contact plane. The dimensionless coordinates of a point situated on area F equals to $x = \hat{x}/\hat{a}$ and $y = \hat{y}/\hat{a}$, where $\hat{x}$ and $\hat{y}$ are the corresponding actual coordinates, while $\hat{a}$ is a radius of real contact.

The following form of non-dimensional contact pressure distribution [5] is assumed:

$$\sigma(x, y) = \hat{\sigma}(x, y)\frac{\hat{a}^2}{\hat{N}} = \frac{3}{2\pi}\sqrt{1 - x^2 - y^2}(1 + d_c x + d_s y) \tag{1}$$

where

$$d_c = d\cos\gamma = d\frac{v_{rx}}{\sqrt{v_{rx}^2 + v_{ry}^2}}$$
$$d_s = d\sin\gamma = d\frac{v_{ry}}{\sqrt{v_{rx}^2 + v_{ry}^2}} \tag{2}$$

In Eqs. (1–2) $\hat{\sigma}(x, y)$ denotes real contact pressure, $\hat{N}$,normal component of the real resultant force loading the contact, d, rolling resistance parameter, γ, angle describing "direction of rolling". The variables v_{rx} and v_{ry} are the components of the non-dimensional "rolling velocity" $\boldsymbol{v}_r = \hat{\boldsymbol{v}}_r/\hat{a} = v_{rx}\boldsymbol{e}_x + v_{ry}\boldsymbol{e}_y$ ($\hat{\boldsymbol{v}}_r$ is the corresponding real vector, $\boldsymbol{e}_\zeta$ is unit vector of axis $_\zeta$).

The model (1) is some artificial modification of the Hertzian stress distribution, moving the centre of the pressure distribution outside the geometrical centre A of the contact and allowing to model the non-dimensional rolling resistance. The latter finally reads

$$\boldsymbol{M}_r = \boldsymbol{f} \times \boldsymbol{e}_z = y_S\boldsymbol{e}_x - x_S\boldsymbol{e}_y = M_{rx}\boldsymbol{e}_x + M_{ry}\boldsymbol{e}_y \tag{3}$$

where